# Viscosity of Suspensions. Finite Size Effects and Polydispersity.

Peter Szabo[1] and Rasmus Hansen[1]

[1] Danish Polymer Centre, DTU Chemical Engineering, Technical University of Denmark
Produktionstorvet, Building 423, DK-2800 Kongens Lyngby, Denmark

## ABSTRACT

The viscosity of particle suspensions depend on the volume fraction of particles. This was demonstrated by Einstein[1,2] for dilute suspensions and later extended to less dilute systems by Batchelor[3] and Batchelor and Green[4]. Krieger and Dougherty[5] followed a different approach to analyse concentrated suspensions. They combined the result of Einstein with a self-consistent field approach and obtained a very useful expression for the viscosity as a function of the solvent viscosity ($\eta_s$) and the volume fraction of particles ($\varphi$) as well as a maximum packing parameter ($\varphi_m$). The maximum packing parameter $\varphi_m$, though physically sound, was introduced in a very heuristic way.

In recent work by Hansen and Szabo[6,7] a model was developed for the viscosity of particle suspensions which takes into account finite size effects. Data reported by de Kruiff et al.[8] were compared to predictions by the new model as well as to other models from the literature (e.g. Krieger and Dougherty[5]). We found that the inclusion of finite size effects improved the model prediction of the viscosity.

The idea of a discrete spectrum of particles was also presented in Hansen and Szabo[6,7]. Here we shall explore the model consequences as the particles in suspension are no longer identical.

## INTRODUCTION

The viscosity of particle suspensions depends on the volume fraction of suspended particles. Based on viscous dissipation by the flow around a single isolated non-deformable sphere, Einstein[1,2] calculated the change in viscosity $\eta$ to first order in the volume fraction $\varphi$.

$$\eta=\eta_s(1+[\eta]\varphi) \tag{1}$$

Here, $\eta_s$ is the solvent viscosity and the intrinsic viscosity $[\eta]=5/2$. In general the intrinsic viscosity is defined by Eq. 1 in the limit $\varphi\rightarrow 0$ and depends on the geometry and deformability of the suspended particles.

## BASIC DISCRETE MODEL WITH FINITE SIZE EFFECTS

We consider here a volume composed of solvent and identical particles so that the total volume equals the sum of solvent and particle volumes $V_s$ and $nV_p$ respectively. Here, $V_p$ denotes the volume of a single particle and n is the number of such particles.

The volume fraction of particles is then calculated from

$$\varphi_n=nV_p/(V_s+nV_p) \tag{2}$$

We may now assume that the solvent including n particles can be considered a continuum fluid and apply the Einstein expression for the viscosity of dilute suspensions when adding particle number n+1. We obtain,

$$\eta_{n+1}=\eta_n+\eta_n[\eta]\Delta\varphi_n^* \quad (3)$$

where $\Delta\varphi_n^*$ is the single particle volume fraction increment

$$\Delta\varphi_n^*=V_p/(V_s+(n+1)V_p) \quad (4)$$

As argued in Hansen and Szabo[6,7] it is reasonable to introduce a maximum packing fraction $\varphi_m$. This way divergence in the viscosity is ensured as $\varphi\rightarrow\varphi_m$. We choose the simple form suggested by Krieger and Dougherty[5] so that the change in viscosity becomes:

$$\eta_{n+1}=\eta_n+\eta_n[\eta]\Delta\varphi_n^* /(1-\varphi_n/\varphi_m) \quad (5)$$

This expression for the viscosity increment was demonstrated[7] to compare well with data from de Kruiff et al.[8] in the continuous limit as $V_p \rightarrow 0$ while the total volume of particles $nV_p$ is kept constant.

POLYDISPERSITY OR MORE THAN A SINGLE PARTICLE SIZE

The model approach described above was quite successful in describing the viscosity of a monisperse suspension with a single partcle size. It is possible, however, to describe a suspension that contains more than one characteristic particle size. We may begin the analysis by defining an ensemble of particles identified by a type identifier j=1, 2 ... M with corresponding particle volumes $V_{pj}$ and numbers $n_j$. Then we can calculate the total number of particles

$$N=\sum_j n_j \quad (6)$$

and a total paticle volume

$$V_p=\sum_j n_jV_{pj} \quad (7)$$

From here we continue by defining the total volume fraction of particles

$$\varphi=\sum_j \varphi_j =\sum_j V_{pj}/(V_s+V_p) \quad (8)$$

If we assume that different particle sizes require different maximum packing fractions we may then derive expressions similar to equations (2)-(5).

REFERENCES

1. Einstein, A. (1906), "Effect of Suspended Rigid Spheres on Viscosity", *Ann. Phys.*, **19**, 289-306.

2. Einstein, A. (1911), "Berichtigung", *Ann. Phys.*, **34,** 591-592.

3. Batchelor, G.K. (1970), "The Stress System in a Suspension of Force-free Particles", *J. Fluid Mech.*, **41,** 545-570.

4. Batchelor, G.K. and Green, J.T. (1972), "The Determination of the Bulk Stress in a Suspension of Spherical Particles to Order $c^2$ ", *J. Fluid Mech.*, **56**, 401-427.

5. Krieger, I.M. and Dougherty, T.J. (1959), "Concentration Dependence of the Viscosity of Suspensions", *Trans. Soc. Rheol.*, **3,** 137-152.

6. Hansen, R. and Szabo, P. (2008), "Viscosity of Suspensions and Finite Size Effects", *Ann. Trans. Nordic Rheol. Soc.*, **16**, 123-124.

7. Hansen, R. and Szabo, P. (2009), "Finite Size Particle Dependence of the Viscosity of Suspensions", *Rheol. Acta*, s**ubmitted.**

8. de Kruiff, C.G., van Iersel, E.M.F., Vrij, A. and Russel, W.B. (1985), "Hard Sphere Colloidal Dispersions: Viscosity as a Function of Shear Rate and Volume Fraction", J. *Chem. Phys.*, **83,** 4717-4725.