HEAT TRANSFER IN LAMINAR FLOW OF VISCOELASTIC FLUIDS IN STRAIGHT TUBES OF ARBITRARY SHAPE

Dennis A. Siginer[1] and Mario F. Letelier[2]

1 - Department of Mechanical Engineering , Wichita State University, Wichita, KS 67260-0133, USA and Ecole Polytechnique de l'Universite de Nantes, Laboratoire de Thermocinetique-CNRS, France

2- Departamento de Engeniaria Mecanica, Universidad de Santiago de Chile, Santiago, Chile

ABSTRACT

The fully developed thermal field in constant pressure gradient driven laminar flow of viscoelastic fluids in straight pipes of arbitrary contour ∂D is investigated. The nonlinear fluids considered are constitutively represented by a class of single mode, non-affine constitutive equations. The driving forces can be large and inertial effects are accounted for. Asymptotic series in terms of the Weissenberg number *Wi* are employed to represent the field variables. Heat transfer enhancement due to shear-thinning is identified together with the enhancement due to the inherent elasticity of the fluid. The latter is the result of secondary flows in the cross-section. Increasingly large enhancements are computed with increasing elasticity of the fluid as compared to its Newtonian counterpart. Large enhancements are possible even with dilute fluids. Isotherms for the temperature field are presented and discussed for several non-circular contours such as the ellipse and the equilateral triangle together with heat transfer behavior in terms of the Nusselt number *Nu.*

I. INTRODUCTION

Experimental findings concerning heat transfer characteristics of aqueous polymer solutions flowing in straight tubes point at considerable enhancement as compared to its Newtonian counterpart driven by the same conditions and in the same geometry. Specifically, it is reported that heat transfer results for viscoelastic aqueous polymer solutions are considerably higher in flows fully developed both hydrodynamically and thermally, as much as by an order of magnitude depending primarily on the constitutive elasticity of the fluid and to some extent on the boundary conditions, than those found for water in laminar flow in rectangular ducts, Hartnett and Kostic[1,2]. Heat transfer phenomena in laminar flow of nonlinear fluids with the exception of inelastic shear-thinning fluids in tubes of rectangular cross-section has not been the subject of many investigations in spite of the widespread use of some specific contours in industry such as flattened elliptical tubes. This statement rings true for all cross-sectional shapes for both steady and unsteady phenomena including quasi-periodic flows. Heat transfer research with viscoelastic fluids was declared to be a new challenge in the early nineties, Hartnett[3], but progress has been limited since that time. Highly enhanced heat transfer to aqueous solutions of polyacrylamide and polyethylene of the order of 40% to 45% as compared to the case of pure water in flattened copper tubes was observed by

Oliver[4], and later by Oliver and co-workers as early as 1969. Recent numerical investigations in rectangular cross-sections of Gao and Hartnett[5,6], Naccache and Souza Mendes[7], Payvar[8] and Syrjala[9] establish the connection between the enhanced heat transfer observed and the secondary flows induced by viscoelastic effects. The former researchers as well as Naccache and Souza Mendes predict for instance viscoelastic Nusselt numbers as high as three times their Newtonian counterparts. Gao and Hartnett[5] report results for rectangular contours which provide evidence that the stronger the secondary flow (as represented by the dimensionless second normal stress coefficient Ψ_2) the higher the value of the heat transfer (as represented by the Nusselt number *Nu*) regardless the combination of thermal boundary conditions on the four walls. Constant heat flux is imposed everywhere on the heated walls in their experiments with the remaining walls being adiabatic. The combination of boundary conditions also plays some role in the enhancement as reported with the largest enhancement occurring when two opposing walls are heated. Despite these efforts heat transfer characteristics of viscoelastic fluids in steady laminar flow in rectangular tubes remains very much an open question.

It must also be pointed out that the shear rate dependent viscosity of purely viscous fluids (negligible relaxation time) is responsible for enhanced and reduced heat transfer in the case of shear thinning and shear thickening fluids, respectively, as shown by Gingrich et al[10] and others . But there is evidence in the literature that in the case of viscoelastic fluids the effect of the shear rate dependent viscosity on heat transfer enhancement is at least two orders of magnitude smaller when compared to the influence of the secondary flow Naccache and Souza Mendes[7]. Thus the latter remains the dominant mechanism for enhanced heat transfer.

In this paper we investigate the heat transfer behavior of a class of non-affine viscoelastic fluids in straight tubes of non-circular contour in pressure gradient driven laminar flow and under constant temperature wall conditions. Although we work with a specific type of fluid within the family of fluids the results obtained are representative of the behavior of fluids in this class. The solution of the nonlinear field and constitutive equations is obtained via hierarchical regular perturbation problems derived through the expansion of the field variables into asymptotic series in terms of the Weissenberg number *Wi*. The thermal field is solved in tandem with the velocity field. The solution at the zeroth order is the Newtonian field in a straight pipe of non-circular contour, Letelier & Siginer[11,12]. Thus the thermal field at the zeroth order represents the temperature distribution in a Newtonian fluid in a tube of arbitrary contour. Additional longitudinal fields due to viscoelastic effects appear at the second and third orders together with the secondary field at the third order, Letelier & Siginer[11,12]. The thermal field at the first order is null, a consequence of a null first order velocity field. At the second order the longitudinal field is affected by the viscoelastic nature of the fluid, by both shear-thinning and first normal stress effects, and as a consequence the thermal field is altered separately with additive superposed effects by shear-thinning and elasticity. The longitudinal field is further changed at the third order with a corresponding change in the thermal field, but more importantly at this order a secondary flow triggered by unbalanced second normal stresses brings large changes to the temperature distribution and heat dissipation.

II. MATHEMATICAL ANALYSIS

The structure of the class of nonlinear viscoelastic fluids of interest in this work has been described at length in Letelier &

Siginer[13]. A summary is given here for completeness. The family of single mode constitutive structures which relates the deformation measure **D** to the viscoelastic contributed stress tensor τ is defined by

$$f(\varepsilon_{\circ}, \mathrm{tr}\boldsymbol{\tau})\,\boldsymbol{\tau} + \lambda\,\overset{\circ}{\boldsymbol{\tau}} = 2\,\eta_{m}\,\mathbf{D} \qquad (1)$$

through a relaxation time λ, a molecular contributed viscosity η_m and a function f related to the elongational properties of the fluid, both to be defined shortly. The total stress $\boldsymbol{\sigma}$ is written as

$$\boldsymbol{\sigma} = -\mathrm{P}\,\mathbf{I} + 2\eta_{\mathrm{N}}\,\mathbf{D} + \boldsymbol{\tau} \qquad (2)$$

with P and η_N representing the pressure field and the Newtonian viscosity of the continuum, respectively. The class of nonaffine constitutive models represented by (1) includes the Johnson-Segalman and Phan-Thien-Tanner models framed in terms of the Gordon-Schowalter convected derivative $\overset{\circ}{(\bullet)}$,

$$\overset{\circ}{\boldsymbol{\tau}} = \dot{\boldsymbol{\tau}} - \nabla\mathbf{u}^{T}\,\boldsymbol{\tau} - \boldsymbol{\tau}\nabla\mathbf{u} + \xi(\boldsymbol{\tau}\mathbf{D} + \mathbf{D}\boldsymbol{\tau})$$

Where $\dot{(\bullet)}$ indicates the material derivative. The family of constitutive models defined by (1) has two constitutive parameters, ε_o and ξ apart from the relaxation time λ and other constants embedded in the molecular viscosity η_m. ε_o and ξ define the elongational and the non-affine deformation of the material, respectively, as well as the shear-thinning ability of it. The deformation of the macroscopic medium is affine by definition, but polymer strands embedded in the medium may slip with respect to the deformation of the macroscopic medium. Thus each strand may transmit only a fraction of its tension to the surrounding continuum as the continuum slips past the strands. This kind of slippage is taken into account in the Gordon-Schowalter convected derivative by the material parameter ξ. When $\xi = 0$ there is no slippage, the motion becomes affine, the Gordon-Schowalter derivative collapses onto the upper convected derivative, and shear thinning is entirely governed by the material parameter ε_o.

The function $f(\varepsilon_o, \mathrm{tr}\,\tau)$ in (1) may be defined as an exponential function of the material parameter ε as introduced by Xue et al. [14], $f(\varepsilon_o, tr\boldsymbol{\tau}) = \exp\left(\dfrac{\varepsilon_{\circ}\lambda}{\eta_{mo}} tr\,\boldsymbol{\tau}\right)$. (2)

If the trace of τ is small a linear form can be used. The molecular contributed zero shear viscosity η_{mo} is introduced in (2). If $\varepsilon_o = 0$ that is $f = 1$ we obtain a model which can describe the response of fluids to forcing which lead to negligible elongational deformations. In elongational flows stress develops a singularity with $f = 1$ and stress growth in steady-state extension becomes unbounded. The molecular viscosity η_m which appears in (1) is defined in terms of the shear rate $\kappa = (2\,tr\,\mathbf{D}^2)^{1/2}$, λ, ξ and η_{mo},

$$\eta_m = \quad \eta_{mo}\frac{1 + \xi(2-\xi)\lambda^2\kappa^2}{(1 + \lambda^2\kappa^2)^{(1-m)/2}} \;.$$

The power-law index m is always $m \leq 1$. The zero-shear rate viscosity of the fluid is defined as, $\eta_o = \eta_{No} + \eta_{mo}$. η_{No} is the zero-shear rate viscosity of the Newtonian solvent. Total stress (2) is rewritten as,

$$\boldsymbol{\sigma} = -\mathrm{P}\,\mathbf{I} + 2\eta_o(1-\beta)\,\mathbf{D} + \boldsymbol{\tau}\,,\ \beta = \frac{\eta_{mo}}{\eta_o} \quad .$$

Setting β equal to one yields a fluid, whose total viscosity is contributed by the long chain molecules only. Further setting m = 1 one obtains,

$$\eta_o = \eta_{mo} = \eta = cte\,, \qquad \eta_m = \beta\,\eta\,\mu = \eta\,\mu_o$$

$$\mu_o = 1 + \lambda^2\xi(2-\xi)\kappa^2$$

We note that setting $\mu_o = 1$ will not lead by itself to a prediction of rectilinear flow in tubes of non-circular contour but setting $f(\varepsilon_{\circ}, \mathrm{tr}\boldsymbol{\tau}) = 1$ will no matter the value assigned to μ_o as the apparent viscosity and the second normal stress coefficient become proportional. If both $\mu_o = 1 = f(\varepsilon, \mathrm{tr}\boldsymbol{\tau})$ the apparent viscosity and the second normal stress coefficient become constants $\eta(\kappa^2)=\eta_o,\ \ \psi_1(\kappa^2)=2\lambda\eta_o,\ \ \psi_2(\kappa^2)=-\xi\lambda\eta$ and no secondary flows can be predicted.

II.1 Field equations

The balance equations read as,

$$\rho\, u_i u_{j,i} = \sigma_{ji,i} \qquad u_{i,i} = 0$$

$$\sigma_{ij} = -P\delta_{ij} + \tau_{ij}$$

Where ρ and P are scalar parameters representing the density and the total pressure field. Setting β equal to one and introducing dimensionless variables,

$$r^* = \frac{r}{a},\ z^* = \frac{z}{a},\ u^* = \frac{u}{V_0},\ v^* = \frac{v}{V_0},\ w^* = \frac{w}{V_0},$$

$$P^* = \frac{a\,P}{\eta_{m0} V_0},\ \tau^*_{ij} = \frac{a\,\tau_{ij}}{\eta_{m0} V_0},\ D^*_{ij} = \frac{a}{V_0} D_{ij}$$

based on the molecular contributed zero shear viscosity η_{m0} (assuming $\eta_{N0} \sim 0$), a characteristic velocity V_0 and a characteristic length a, and adopting the linear form of $f^*(\varepsilon, \mathrm{tr}\,\tau^*_{ij})$, the constitutive and balance equations are rewritten in dimensionless form in a cylindrical frame with the velocity components **u** = (u, v, w)

$$2\mu_0\, D^*_{ij} = f^*(\varepsilon_\circ, \mathrm{tr}\,\tau^*_{ij})\,\tau^*_{ij} + Wi\,\overset{\circ}{\tau}{}^*_{ij},$$

$$2\left[1 + 2\,\xi\,(2-\xi)\,Wi^2 tr\, D^{*2}_{ij}\right] D^*_{ij} = (1 + \varepsilon_\circ\, Wi\; tr\,\tau^*_{ij})\,\tau^*_{ij} + Wi\,\overset{\circ}{\tau}{}^*_{ij},$$

$$u^* u^*_{,r^*} + \frac{v^*}{r^*} u^*_{,\theta} + w^* u^*_{,z^*} - \frac{v^{*2}}{r^*} = F^*_r - P^*_{,r^*} + \left[\nabla^{*2} u^* - \frac{u^*}{r^{*2}} - \frac{2}{r^{*2}} v^*_{,\theta}\right]$$

$$u^* v^*_{,r^*} + \frac{v^*}{r^*} v^*_{,\theta} + w^* v^*_{,z^*} + \frac{u^* v^*}{r^*} = F^*_\theta - \frac{1}{r^*} P^*_{,\theta} + \left[\nabla^{*2} v^* - \frac{v^*}{r^{*2}} + \frac{2}{r^{*2}} u^*_{,\theta}\right]$$

$$u^* w^*_{,r^*} + \frac{v^*}{r^*} w^*_{,\theta} + w^* w^*_{,z^*} = F^*_z - P^*_{,z^*} + \nabla^{*2} w^*.$$

$$\text{Weissenberg number} = Wi = \frac{\lambda\, w_0}{a}.$$

F_r, F_θ and F_z represent the viscoelasticity contributed force components in the momentum balance,

$$F^*_r = (\nabla^* \bullet \boldsymbol{\tau}^*)_{r^*} = \frac{1}{r^*}\left(r^*\,\tau^*_{r^*r^*}\right)_{,r^*} + \frac{1}{r^*}\,\tau^*_{r^*\theta,\theta} - \frac{\tau^*_{\theta\theta}}{r^*} + \tau^*_{z^*r^*,z^*},$$

$$F^*_\theta = (\nabla^* \bullet \boldsymbol{\tau}^*)_\theta = \tau^*_{r^*\theta,r^*} + \frac{1}{r^*}\,\tau^*_{\theta\theta,\theta} + \frac{2}{r^*}\tau^*_{r^*\theta} + \tau^*_{\theta z^*,z^*},$$

$$F^*_z = (\nabla^* \bullet \boldsymbol{\tau}^*)_{z^*} = \frac{1}{r^*}(r^*\,\tau^*_{r^*z^*})_{,r^*} + \frac{1}{r^*}\,\tau^*_{\theta z^*,\theta} + \tau^*_{z^*z^*,z^*}.$$

The heat diffusion equation defines the thermal field,

$$\frac{DT}{Dt} = \alpha\nabla^2 T,$$

$$u\,T_{,r} + \frac{v}{r}\,T_{,\theta} + w\,T_{,z} = \alpha\left[T_{,rr} + \frac{1}{r}\,T_{,r} + \frac{1}{r^2}\,T_{,\theta\theta} + T_{,zz}\right].$$

Where α is the thermal diffusivity. The dimensionless temperature field T^* is introduced

$$T^*(r,\theta,z) = \frac{T(r,\theta,z) - T_w(z)}{T_m(z) - T_w(z)}.$$

T_W and T_m denote the constant wall temperature and the average temperature, respectively, with $V_m = \frac{1}{A}\iint_s w\,ds$

$$T_m = \frac{1}{AV_m}\iint_s wT ds\ ,\ A = \iint_s ds\ .$$

We consider the case of a constant heat flux along the pipe wall, that is

$$q_w'' = h(T_m - T_w) = cte.$$

The temperature at any given cross-section is constant in time, but there is a longitudinal temperature gradient. The dimensionless heat diffusion equation reads as

$$T^*_{,r^*r^*} + \frac{1}{r^*}T^*_{,r^*} + \frac{1}{r^{*2}}T^*_{,\theta\theta} =$$

$$Pr\left[u^* T^*_{,r^*} + \frac{v^*}{r^*}T^*_{,\theta} + w^* T^*_{m,z^*}\right]$$

Where *Pr* stands for the Prandtl number, and the dimensionless average temperature gradient a_0 has been introduced,

$$Pr = \frac{\eta}{\rho\alpha}, \qquad a_0 = T^*_{m,z^*} = \frac{1}{\Delta T}\frac{\partial T_m}{\partial z^*},$$

$$\Delta T = T_m(z) - T_w(z).$$

The star notation is omitted from here on, and unless noted otherwise the variables and the expressions used are dimensionless.

II.2 Solution of the field equations

The balance and constitutive equations are solved by expanding the field variables in power series in terms of the Weissenberg number *Wi*. The expansions in power series of the field variables

$$\Im = \sum_{m=0}^{\infty} Wi^m\, \Im^{\langle m\rangle},\ \Im = (u, v, w, \boldsymbol{\tau}, \varphi, P, \mathbf{D}, \boldsymbol{\sigma})$$

are used in the linear momentum and the non-linear constitutive stress-strain relationship to obtain hierarchical regular perturbation problems.

II.2.1 Fields at order $\boldsymbol{O}(1)$:

$$\mathbf{u}^{\langle 0\rangle} \bullet \nabla \mathbf{u}^{\langle 0\rangle} = \nabla \bullet \boldsymbol{\sigma}^{\langle 0\rangle}, \qquad \nabla \bullet \mathbf{u}^{\langle 0\rangle} = 0\ ,$$

$$\boldsymbol{\sigma}^{\langle 0\rangle} = -P^{\langle 0\rangle}\mathbf{I} + \tau^{\langle 0\rangle},$$

$$\mathbf{u}^{\langle 0\rangle} = (0, 0, w^{\langle 0\rangle})$$

$$\boldsymbol{\tau}^{\langle 0\rangle} = (\mathbf{e_r}\otimes\mathbf{e_z} + \mathbf{e}_z\otimes\mathbf{e_r})\, w^{<0>}_{,r} +$$

$$(\mathbf{e}_\theta\otimes\mathbf{e_z} + \mathbf{e}_z\otimes\mathbf{e}_\theta)\frac{1}{r} w^{<0>}_{,\theta}$$

$$\nabla^2 w^{\langle 0\rangle} = \frac{1}{2}P^{<0>}_{,z} = -4p = cte.$$

The solution which satisfies mass conservation is given by,

$$w^{\langle 0\rangle} = p\left[1 - r^2 + \varepsilon r^n \sin(n\theta)\right] = p\, w_0\,,$$

$$0 < \varepsilon < 1, \quad n > 1. \tag{5}$$

We note that $r^n \sin(n\theta)$ as well as $r^n \cos(n\theta)$ are part of the infinite set of homogeneous solutions of the Laplace equation and we could as well have used a superposition

$$w = p\left[1 - r^2 + \sum_{n=1}^{N}\varepsilon_n r^n \begin{Bmatrix}\sin(n\theta+\theta_n)\\ \cos(n\theta+\theta_n)\end{Bmatrix}\right]$$

$$n = 1, 2, 3, \ldots, N, \tag{6}$$

where a constant phase angle θ_n has been introduced. We note that to satisfy the no-slip condition on the contour ∂D we must have

$$w^{\langle 0\rangle}\Big|_{\partial D} = 0, \qquad 1 - r^2 + \varepsilon r^n \sin n\,\theta = 0. \tag{7}$$

Equation $(7)_2$ is called the shape factor and defines a mapping of the basic circular cross-sectional shape into various non-circular shapes through the parameters ε and n. Varying the parameters continuously in the ranges indicated in (5) correspond to a continuous deformation of the circular contour. For instance n=4, ε=0.22 corresponds to a square. A sharp right angle corner is not possible to attain exactly as a curvature however very small is required at the corner for continuity. But from a practical point of view this does not carry any importance as the corner can be made indeed quite close to sharp by varying ε. The phase angle θ_n leads to a rotation of the mapped contour in the (r,θ) plane. Equation (6) may be conceived of as the superposition of a finite number of solutions of type (5), each one rotated by a certain angle with respect to the others. Thus it is possible to superpose two or more mapped figures such as an ellipse (n=2, $\varepsilon_c > \varepsilon > 0$) and a triangle (n=3, $\varepsilon_c > \varepsilon > 0$) either one rotated by a certain angle with respect to the other to obtain say tear drop shaped contours commonly used in extruders. If ε exceeds a critical value ε_c introduced in the next paragraph the mapping is no longer a closed curve. Note that for n=2 and n=3 and different numerical values assigned to ε one gets ellipses with different aspect ratios, and triangles with varying degrees of curvature of the sides and in particular varying degrees of sharpness at the corners. n=3 and ε=0.385 yields a triangle with straight sides and sharp corners.

Shape factor may not yield closed curves for arbitrarily assigned pair of values for (ε, n). For closed curves the value of ε can not exceed an upper limit $\varepsilon_c = f(n)$. The latter comes out of the requirement that at a cusp the velocity gradient should be zero:

$$\varepsilon_c = \frac{2}{n}\left(\frac{n-2}{n}\right)^{(n-2)/2} .$$

In practice ε assumes fractional values and admissible closed form shapes are given only by integer values of n. Assuming that the boundary ∂D_0 of the domain D_0 of the flow is a circle, a continuous deformation of the circle is observed with increasing values of $\varepsilon > 0$ for a fixed integer n up to a limiting closed boundary with n sharp corners or cusps obtained when ε is equal to a critical value ε_c.

The extension of these concepts to unsteady flows is not straightforward. For unsteady flows in conduits of unconventional shape the following ansatz can be made with ε<1,

$$w = (1^2 - r^2 + \varepsilon r^n \sin n\,\theta)\left[w_0(r,\theta,t) + \varepsilon H_1(r,\theta,t) + O(\varepsilon^2)\right],$$

where $w_0(r, \theta, t)$ is the closed form axial velocity field of the base flow , that is the velocity field in the base domain D_0 corresponding to flow in a tube with a known contour ∂D_0 such as a circle. Determination of the function H_1 yields the first correction to the axial velocity field in the mapped contour ∂D. These ideas were applied recently by Letelier et al.[15] and Siginer and Letelier[16] to the quasi-periodic flow driven by a pulsating pressure gradient of an integral viscoelastic fluid of the fading memory type in straight conduits of arbitrary shape.

The thermal field at this order is defined by

$$\nabla^2 T^{<0>} = Pr\, a_0 w^{<0>} = Pr\, a_0\, p\left[1 - r^2 + \varepsilon r^n \sin(n\theta)\right]$$

$$T^{<0>}(r,\theta) = \frac{Pr}{16} a_0\, w^{<0>}\left[r^2 - 3 + \varepsilon\frac{(n-3)}{(n+1)} r^n \sin(n\theta)\right].$$

II.2.2 <u>Temperature field at order</u> $\boldsymbol{O}(Wi)$:

$$u^{<1>} = v^{<1>} = w^{<1>}, T^{<1>} = 0 .$$

II.2.3 <u>Temperature field at order</u> $\boldsymbol{O}(Wi^2)$:

$$u^{<2>} = v^{<2>} = 0.$$

$$w^{<2>}(r,\theta) = \xi\,(2-\xi)\, p^3 w_0\left[1 + r^2 - \varepsilon\frac{(n^2+2n-1)}{(n+1)} r^n \sin(n\theta)\right].$$

$$T^{<2>}(r,\theta) = Pr\,\frac{\xi(2-\xi)}{36}\, p^3 a_0\, w_0\left\{\begin{array}{l}\varepsilon\, r^n \sin(n\theta)\left[\begin{array}{l}-\dfrac{(7n^2+21n-4)}{(n+2)(n+1)}\\ (1+r^2) + \dfrac{2(10n^2+11n-17)}{(n+1)^2}\end{array}\right]\\ +2r^4 + 2r^2 - 16\end{array}\right\}$$

II.2.4 T<u>emperature field at order</u> $\boldsymbol{O}(Wi^3)$:

The defining equation for the longitudinal velocity at this order is:

$$u^{<3>} w^{<0>}_{,r} + \frac{v^{<3>}}{r} w^{<0>}_{,\theta} = -P^{<0>}_{,z} + 2\nabla^2 w^{<3>} +$$

$$(\nabla \bullet \overset{o}{\boldsymbol{\tau}}{}^{<2>})_z - \varepsilon_0 \left[\nabla \bullet (tr\boldsymbol{\tau}^{<2>})\boldsymbol{\tau}^{<0>}\right]_z$$

It turns out that the contribution of the last term in the above equation is zero regardless whether $\mu_o = 1$ or not, that is the longitudinal field is not affected by the constitutive constant ε_o up to and including the third order in this analysis.

As in the previous orders we introduce a streamfunction for the transversal velocity field and derive for the PTT fluid:

$$r\nabla^4 \psi^{<3>} = \frac{\partial}{\partial r}(rF_\theta^{<3>}) - \frac{\partial F_r^{<3>}}{\partial \theta} =$$

$$8\varepsilon\xi^2(2-\xi)p^4 n(n+4)(n-1)r^{n+1}\cos(n\theta)$$

$$\psi^{<3>}(r,\theta) = \frac{1}{4}\varepsilon\xi^2(2-\xi)p^4\left[1-r^2+\varepsilon r^n \sin(n\theta)\right]^2$$

$$\frac{n(n-1)(n+4)}{(n+1)(n+2)} r^n \cos(n\theta)$$

The solution for $w^{<3>}$ is,

$$w^{<3>}(r,\theta) = \frac{1}{32}\varepsilon\xi^2(2-\xi)p^5 w_0 \frac{n^2(n+4)(n-1)}{(n+1)^2(n+2)^2}$$

$$\left[(n+1)r^2-(n+3)\right] r^n \sin(n\theta)$$

.

The thermal field at this order is given by

$$\nabla^2 T^{<3>} = Pr\left[u^{<3>}T^{<0>}_{,r} + \frac{v^{<3>}}{r}T^{<0>}_{,\theta} + a_0 w^{<3>}\right]$$

$$T^{<3>}(r,\theta) = \varepsilon\, w^{<0>} Pr^2 \,\frac{\xi^2(2-\xi)}{384} p^4 a_0$$

$$\left(\frac{n^2(n+4)(n-1)}{(n+1)^2(n+2)^2}\right) r^n \sin(n\theta)$$

$$\left\{\begin{array}{l} 2(n+1)r^4-(7n+13)(1+r^2)+ \\ \dfrac{6(2n^2+7n+7)}{(n+1)} \end{array}\right\}$$

II.3 Computation of the Nusselt Number

The Nusselt number Nu is defined as,

$$Nu = \frac{D_h\ h}{k} = D_h \frac{\text{Grad}}{(T_w - T_m)}$$

$$D_h = \frac{4A}{P} \qquad \text{Grad} = -\frac{1}{P}\int(\nabla T \bullet \mathbf{n})\,dl$$

D_h denotes the effective or hydraulic diameter defined in terms of the area A and the perimeter P of the cross section. The average temperature gradient, the wall temperature, the average temperature and the outward unit normal vector are represented by "Grad", T_w, T_m and **n**, respectively.

CONCLUSIONS

Inertial as well as elastic effects in the laminar flow of a class of non-affine viscoelastic fluids have been investigated in pressure gradient driven flow in straight tubes of non-circular shape. An asymptotic solution of the field and constitutive equations is obtained via a regular perturbation solution with the Weissenberg number *Wi* as the parameter. Unconventional cross-sectional shapes result from a continuous one-to-one mapping taking the circle into the desired shape such as the ellipse, the triangle and the square. The longitudinal field is determined up to and including the third order in *Wi.* Heat transfer due to shear thinning controlled by the slippage parameter is identified as well as the heat transfer enhancement due to secondary flows. Examples of total heat transfer enhancements for an ellipse and a triangle are given in the following figures with corresponding secondary flow patterns. Details of the computations can be found in the forthcoming publications by Siginer and Letelier[12, 13].

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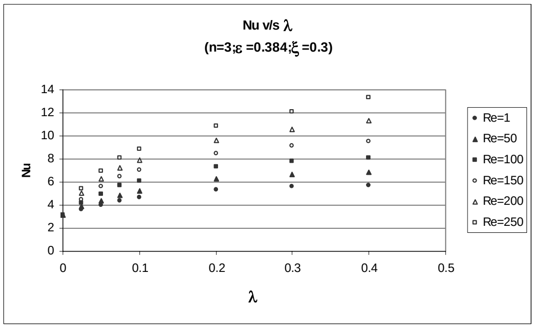

Figure 1. Nusselt number *Nu* versus Weissenberg number *Wi* :equilateral triangle ξ=0.3, n=3, ε=0.384

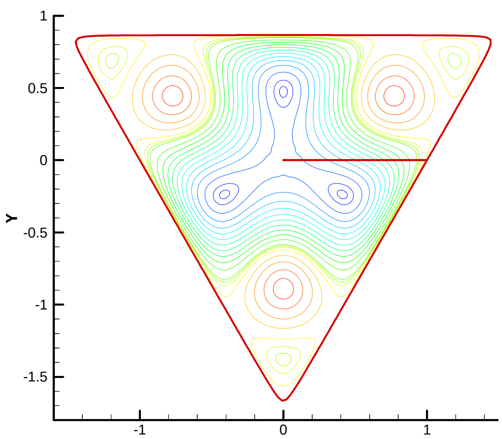

Figure 2. Isotherms : *Re*=200, *Pr*=50, *Wi*=0.3, ξ=0.3

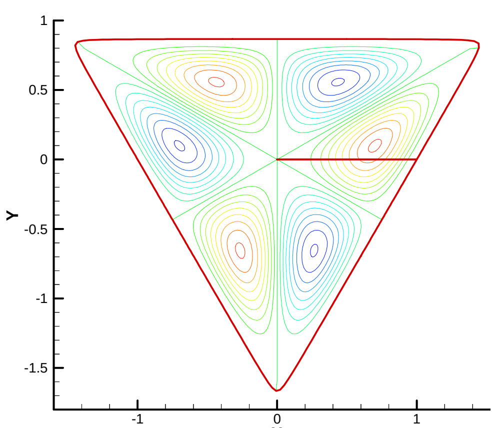

Figure 3. Secondary flows *Re*=200, *Pr*=50, *Wi*=0.3, ξ=0.3

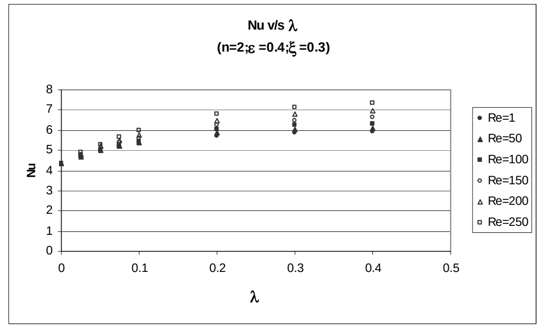

Figure 4. Nusselt number *Nu* versus Weissenberg number *Wi* - ellipse ξ=0.3, n=2, ε=0.4

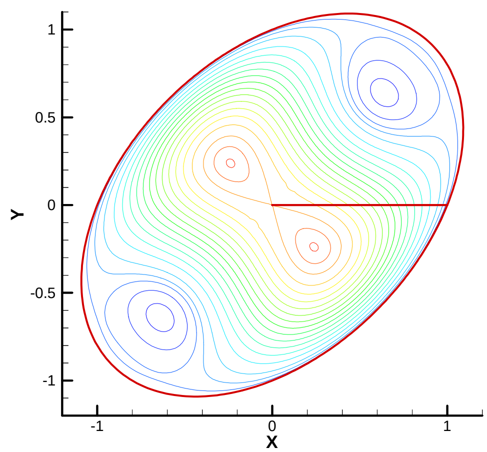

Figure 5. Isotherms : *Re*=200, *Pr*=50, *Wi*=0.3, ξ=0.3

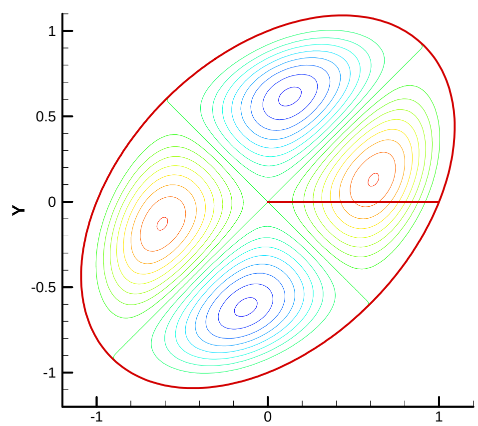

Figure 6. Secondary flows *Re*=200, *Pr*=50, *Wi*=0.3, ξ=0.3

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