Extensional Flow through Hyperbolic Contraction Studied Both Numerically and Experimentally

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ABSTRACT

A hyperbolic contraction technique, specially aimed at the medium viscosity range, is presented and evaluated numerically. Experimental measurements of a Boger fluid have been compared to numerical simulations of corresponding mathematical models.

Enhanced pressure drop for increasing Deborah numbers was observed for both experimental and numerical results.

INTRODUCTION

To govern the properties of viscoelastic fluids both shear and elongational behaviour need to be considered. Since many food products, biomaterials and cosmetics are non-Newtonian, viscoelastic fluids, not only shear viscosity determining techniques are required, but also reliable and effective characterisation techniques for elongational properties. Measuring elongational properties is complicated and therefore only a few techniques are available commercially. These fluids have a medium viscosity range (in-between polymer melts and diluted fluids) and the available techniques are mostly not suitable for this range.

Here, the hyperbolic contraction technique by Wikström and Bohlin [1] and Stading and Bohlin [2,3] has been evaluated as a suitable measuring system for medium viscosity range fluids. The principal of the method is based on contraction flow, where the fluid is pushed through a hyperbolical nozzle at constant displacement speed and the exerted force is measured by a load cell. Samples ranging from doughs to liquid sauces can be tested

GOVERNING EQUATIONS

An isothermal, incompressible viscoelastic fluid in motion is described by the constitutive equations of continuity and motion. They are expressed as follows:

$$\nabla \cdot u = 0 \tag{1}$$

$$Re\left(\frac{\partial u}{\partial t} + u \cdot \nabla u\right) = -\nabla p + \nabla \cdot (\tau + 2\beta D) \tag{2}$$

Here, u is the velocity vector, t the time, p the hydrodynamic pressure, D the rate-of-deformation $(\nabla u + \nabla u^T)/2$ and τ the total extra stress tensor. β is the solvent-viscosity ratio parameter:

$$\beta = \eta_s/(\eta_s + \eta_p), \tag{3}$$

η_s is the solvent viscosity and η_p is the polymeric viscosity. The non-dimensional Re-number is defined as:

$$Re = \frac{\rho U_{avg} L}{\eta_s + \eta_p}, \tag{4}$$

where ρ, L and U_{avg} are the density, the contraction gap width and the characteristic velocity (average upstream velocity) respectively.

The Deborah number, defined as the characteristic relaxation time of the fluid (λ) divided by the characteristic flow timescale is a non-dimensional measure of elastic effects. For this specific configuration the Deborah number for the experimental measurements was calculated as:

$$De = \lambda v / R_0, \tag{5}$$

where v is the flow rate and R_0 is the inlet radius of the configuration. The following correction was used for the pressure drop over the contraction:

$$epd = \frac{(\Delta p - \Delta p_{fd})_B}{(\Delta p - \Delta p_{fd})_N}, \tag{6}$$

where Δp_{fd} is the pressure at fully developed flow for a Boger fluid (B) and a Newtonian fluid (N).

For representing shear ($\dot{\gamma}$) and strain rate ($\dot{\varepsilon}$) use was made of two rate-of-strain invariants referred to as:

$$\dot{\gamma} - 2\sqrt{II_d} \tag{7}$$

$$\dot{\varepsilon} = 3III_d / II_d \tag{8}$$

NUMERICAL METHOD

In this study, numerical simulations of viscoelastic Boger fluids through a hyperbolic contraction configuration were considered. Boger fluids were modelled by the FENE-CR mathematical model [4], which display a constant shear viscosity and an almost quadratic first normal stress difference (N_1). The plateau of extensional viscosity is controlled by the model parameter L, which was set to 5. The Deborah number was varied between De=0.1-50.

The numerical scheme used in this study to gain an approximate solution to the governing equations (1) and (2) is based on a combination of the finite element and the finite volume method (fe/fv(sc)), based on a Taylor-Galerkin[5] and a pressure-correction scheme[6,7].

The flow domain of the axisymmetric hyperbolic contraction used for the numerical analysis is discretised into smaller elements forming a mesh presented in Fig. 1.

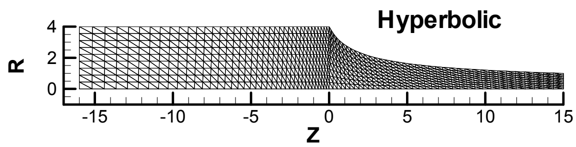


Figure 1. The triangular mesh used to model the fluid domain of the hyperbolic geometry.

Due to symmetry, only half of the flow domain is used for the simulations. No-slip boundary conditions were imposed on the wall, a pressure-driven Poiseuille flow was prescribed at the inlet and a natural streamwise (open) boundary condition was imposed at the exit. The pressure at the exit was set to be zero and all solutions were studied under steady-state conditions.

EXPERIMENTAL METHOD

A schematic drawing of the hyperbolic contraction flow device is shown in Fig. 2.

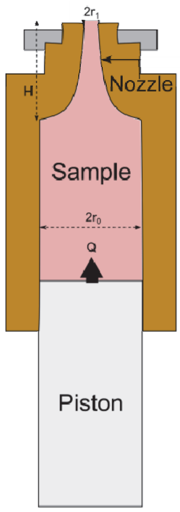


Figure 2. A schematic drawing of the hyperbolic contraction device.

By changing the nozzle, various contraction ratios can be obtained. A piston drives the sample fluid through an axisymmetric hyperbolic contraction at a constant volume flow rate (*Q*). At the start of the measurements the sample goes through a stress growth period before reaching a steady state. The contraction nozzle rests on a load cell and the exerted force on the load cell is measured to determine the extensional properties of the test sample. The device is mounted in an Instron Universal Testing Machine model 5542 (Instron Corporation, Canton, USA) with a measuring range of 500 N.

The viscoelastic Boger fluid used in this experiment consists of 500 ppm polyacrylamide (92560-10G PAA, Sigma-Aldrich Co., USA) dissolved in 6% deionised water and syrup (Dan Sukker, Danisco Sugar, Oslo). A Newtonian fluid with comparable viscosity was also prepared from 6 % deionised water in syrup.

The sample fluid was completely filled into the sample tube with the attached contraction nozzle and allowed to rest for at least 5 min before the start of the measurement to attain a relaxed sample. The temperature was kept constant at T=20°C by a temperature jacket, which was coupled to a controlled circulator Julabo Model FP40 (Julabo Labortechnik, Seelbach, Germany). The measurements were performed at different strain rates at piston rates ranging from 0.5 mm/s to 15 mm/s and reproduced three times to check reproducibility.

A normalised pressure drop for the Boger fluid was calculated by dividing the Boger fluid results with the Newtonian fluid results.

RESULTS AND DISCUSSION

The rheological properties of the test fluids are represented in Fig. 3 and Table 1. The shear viscosities are nearly constant and matched to be the same in order to isolate elastic effects. The first normal stress difference coefficient displays elastic behaviour and shear-thins monotonically.

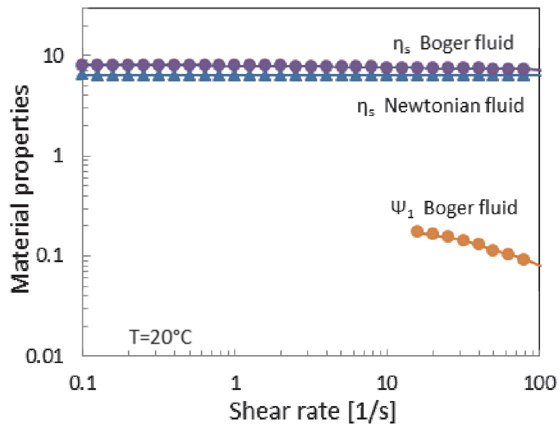


Figure 3. Measurements of shear viscosity (η_s) and first normal stress difference coefficient (Ψ_1) for the Boger and Newtonian test fluid.

Table 1. Rheological properties of the test fluids used.

Rheological properties		
Model fluid	**Zero Shear viscosity η [Pas]**	**Relaxation time λ [s]**
Newtonian fluid	6.4	0
Boger fluid	6.4	0.046

The data in Fig. 4 include normalised pressure drop (*epd*) of experimental measurements (*solid line*) and numerical simulation (*dashed lines*). Under increasing levels of elasticity (*De*) both experimental Boger fluid and numerical FENE-CR results in Fig. 4 show a rise in *epd*.

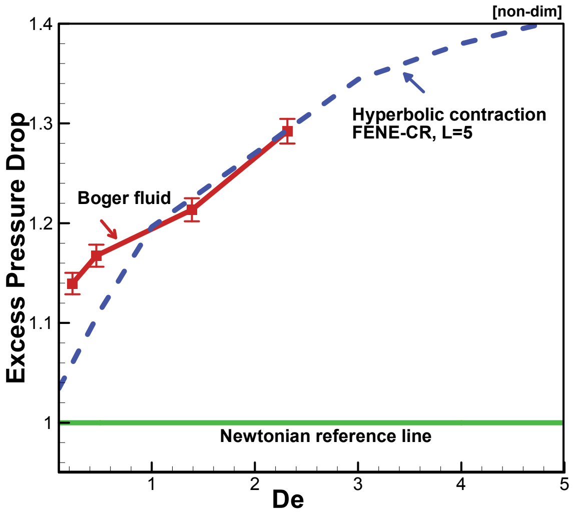


Figure 4. The normalised pressure drop for experimental (solid line) and numerical results (dashed line) are compared for increasing elasticity (De).

Hence, the FENE-CR model is able to detect the behaviour of the Boger fluid through the hyperbolical contraction. The higher the elasticity, the higher the pressure drop. Hence, the pressure drop is a practical measuring parameter for elasticity.

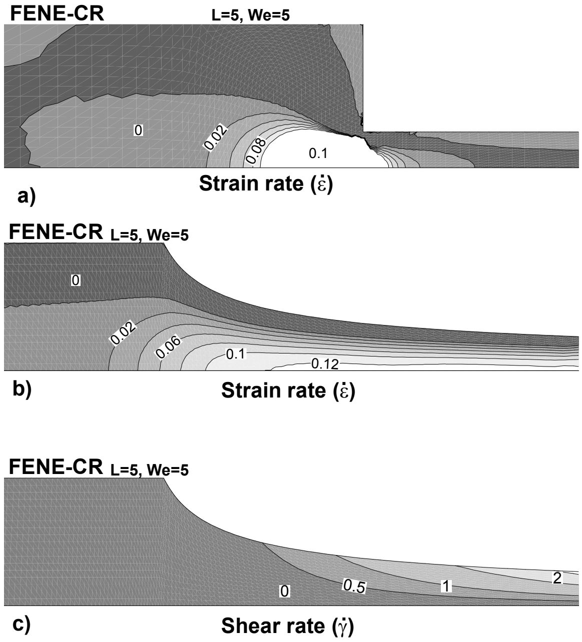
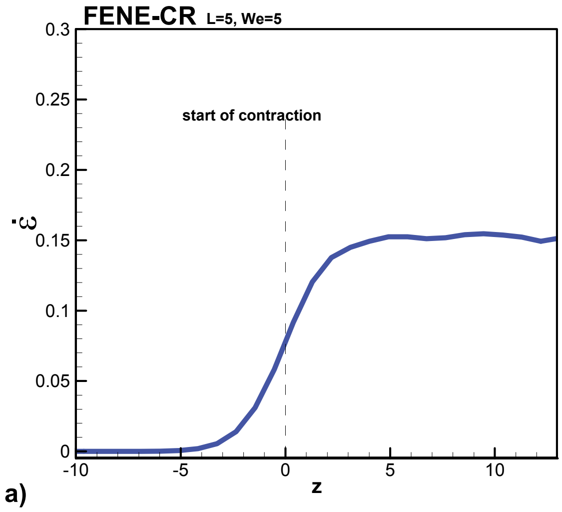


Figure 5. Contour plots of strain rate through a *a)* sharp corner configuration, *b)* hyperbolic configuration. *c)* Contour plot of shear rate through a hyperbolic contraction.

Contour plots of the dimensionless strain rate ($\dot{\varepsilon}$) through both a hyperbolic and a sharp 90 degree configuration are shown in Fig. 4 a-b). One can observe that the area of high extension in the sharp contraction (Fig. 5a) is concentrated around the region of the contraction. In contrast, a uniform, uniaxial extension along the flow centrelines is achieved in the hyperbolic configuration in Fig. 5b. Further validation of this can be seen in Fig. 6a, where the strain rate along the symmetry line is shown. The strain rate is rising near the start of the contraction (z=0), reaching a plateau at z=4 throughout the contraction range. These results confirm the assumption of a uniform extensional flow at the centre throughout the whole contraction.



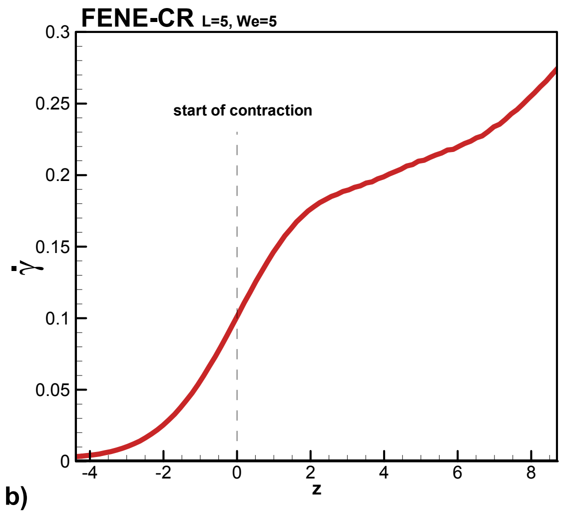


Figure 6. *a)* Strain rate along the symmetry line and b) shear rate along the wall for the hyperbolic configuration.

Though, for a Boger fluid with no shear-thinning properties an influence of shear has to be encountered. The level of shear flow is shown along the configuration wall in Fig. 6b and contour shear rate fields are shown in Fig. 5c. Increasing influence of shear is observed towards the end of the contraction, influencing the measured pressure drop. As a consequence, the epd measures the shear rate contribution as well.

For future work, replacing the contraction die with a contraction-expansion configuration in the measuring system would resolve this issue. The shear contribution to the epd would cancel out resulting in lower epd values and more accurate measurements of extensional properties and.

CONCLUSIONS

By using a hyperbolic configuration instead of a sharp 90 degree configuration the shear contribution is minimized. Further reduction could be achieved by using a contraction-expansion configuration, which is an important improving step for future optimisation of the measuring instrument.

The FENE-CR model has proved to be a suitable model to simulate the flow behaviour of Boger fluids in this work and can further be used to determine rheological functions affecting the flow behaviour of different fluids. The epd value is a practical way to determine extensional fluid properties in the hyperbolic configuration geometry.

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