

## On the Relevance of Laboratory Scale Rheometric Measurements for Calculation of Complex Large Scale Flows in Well Drilling and Pipe Flows

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### ABSTRACT

Accurate measurement of liquid rheological parameters is becoming an invaluable standard test for industrial flow applications. Such measurements are also important to determine non-Newtonian fluid behaviour and even for understanding the physics and structure of fluids.

An important question however is - to what extent laboratory scale rheological data can support realistic prediction of flow in large scale complex geometries, including non-steady motion and often under turbulent conditions. The aim of this paper is to demonstrate by simple experiments some of the challenges that are faced when rheological properties are to be implemented into modelling of such flows.

In this study experiments have been carried out in the vertical pipes of a U-tube rig with oscillatory motion, in order to study such relations in a simplistic way. A numerical simulator was written in Matlab to compare experiments with theory and to carry our parametric studies. Although carried out in laboratory scale such simple experiments are relevant for large scale flows in pipes in process industries as well as for drilling oil and gas wells.

### INTRODUCTION

Drilling involves the use of drilling mud injected through the drill string. The drilling mud serves several purposes, like pressure

control, lubrication, and cleaning out drilling cuttings particles from the wellbore as described by Saasen et al<sup>1</sup>. However gas-kicks may arise due to drilling into high-pressure gas zones, and in applications of gas enriched mud for so-called underbalanced drilling. In such cases there is a possibility that severe oscillations and even extreme accelerations might take place. An upward moving gas kick bubble will expand due to pressure relief and will accelerate. If free gas is developing in either the drill string or the surrounding annulus, even U-tube oscillations may occur. Under such scenarios shear rate and shear stress can vary substantially over time.

The paper describes oscillating gas-liquid flow in a U-tube rig of 4.5 meter height. It was constructed to allow large gas bubbles to either ascend in an open "riser" pipe, or as in this work, held fixed under a closed valve in one of the vertical pipes of the "U". Using a valve in each of the two riser pipes, a gas bubble can be kept initially pressurized. When the valve in the adjacent pipe is suddenly opened, the gas bubble volume adjusts gradually to the external hydrostatic pressure. The flow oscillates by compression and decompression of the gas bubble until pressure equilibrium is reached by friction losses. The oscillation frequency is mainly connected to mass inertia and to gas compressibility, while amplitude decay can be related to liquid viscosity via friction.

A mathematical model was developed to describe the bubble response and the associated oscillatory liquid flow. Both water and non-Newtonian liquids were used. The model simulations are compared with the experimentally observed motion and to laboratory rheometric measurements.

**THEORY**

Large scale fluid flow during drilling of wells involves a variety of flow components and geometries. The fluid rheology is considered to be of paramount importance to design the drilling operations successfully.

Using the rheological data to get realistic friction calculations depends on having accurate knowledge of well geometry. But even small lateral movements of the drill pipe influence velocity profile and friction substantially. Even more if the fluid has yield stress and thixotropic properties.

Oscillating flows in pipes have been studied by many authors, most notably by Schlichting<sup>2</sup>, Uchida<sup>3</sup>, Clamen & Minton<sup>4</sup>, Zhao<sup>5</sup>, and in the book by Bird et al<sup>17</sup>. The following simplified Navier-Stokes equation is solved:

$$\frac{\partial u}{\partial t} = \frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \left( \frac{\partial^2 u}{\partial t^2} + \frac{1}{r} \frac{\partial u}{\partial r} \right) \quad (1)$$

Here  $\rho$  is the liquid density and  $\nu$  is the kinematic viscosity. With velocity boundary conditions  $u=0$  at  $r=R$ , and a periodic harmonic pressure gradient

$$-\frac{1}{\rho} \frac{\partial p}{\partial x} = K \cos(nt) \quad (2)$$

With  $n$  being the oscillation frequency this gives two extreme solutions, as solved by Uchida<sup>3</sup>. One solution is for very slow oscillations, given by

$$u(r, t) = \frac{K}{4\nu} [R^2 - r^2] \cos(nt) \quad (3)$$

This reduces to the well known Poiseuille solution at steady flow ( $n=0$ ).

The other extreme is for very high frequencies  $n$ , given by the following set of equations

$$u(r, t) = \frac{K}{n} \cdot F1 \cdot F2 \quad , \text{ where}$$

$$F1 = \sin(nt) - \sqrt{\frac{R}{r}} \exp\left(-\sqrt{\frac{n}{2\nu}} \cdot (R - r)\right)$$

$$F2 = \sin\left[nt - \sqrt{\frac{n}{2\nu}} \cdot (R - r)\right]$$

..... (4)

This is described in detail in Schlichting<sup>2</sup>, and Libii<sup>6</sup>, who cite Uchida<sup>3</sup> for the following Fig.1. Both solutions are in Fig.2.

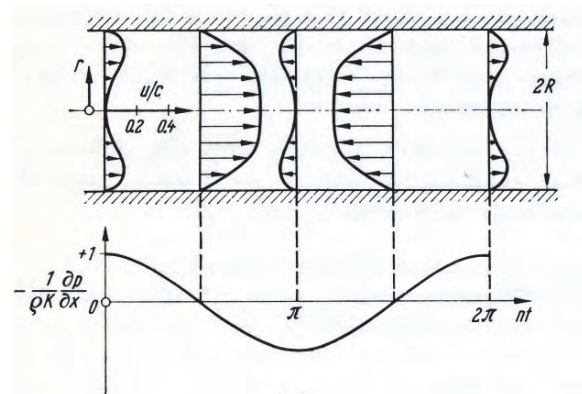
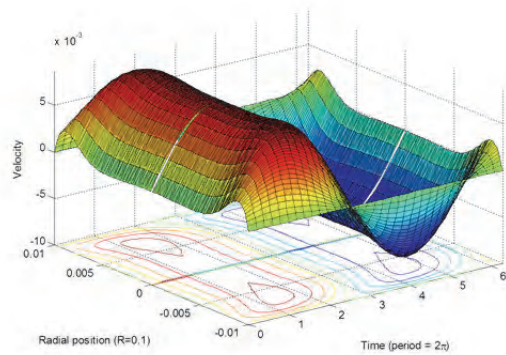
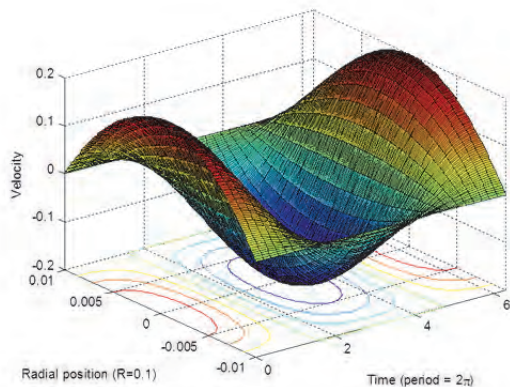


Figure 1. Velocity profile in pipe flow exposed to a bidirectional fast oscillation.

It can be seen - as is also commented by Zhao<sup>5</sup> - that the boundary layer is stable during the accelerating phase of the oscillation, and becomes unstable when the flow returns. Also one may note the overshoot of the velocity profile near the wall at extremes of the pressure gradient which is often referred to as the Richardson annular effect Camacho<sup>6</sup>. Note also the phase difference between pressure gradient and flow velocity for the high frequency limit. In view of the very different velocity profiles in these two cases it is also natural to conclude that fluids with different rheological parameters will respond differently during the oscillation.



a)



b)

Figure 2. Recalculated original solution from Uchida<sup>3</sup>, showing one period of the oscillation.  
 a) The solution for high frequency oscillations.  
 b) The limiting low frequency solution.  
 The pressure gradient is maximum at time  $t=0$ , and it is zero at  $t=\pi/2$  in both plots.

Some authors, Park<sup>8</sup> and Ogawa<sup>9</sup>, also suggest the use of U-tube oscillations to measure rheological parameters.

### Non-Newtonian flows

In the case of non-Newtonian flows, analytical mathematical solutions to the velocity profile cannot be obtained. Kekalov<sup>13</sup> and Bird et al<sup>16</sup> give some analytical solutions. Mostly for power-law fluids as used in the experiments here with CMC it is necessary to use numerical CFD tools. In any case direct measurement using ultrasonics [Takeda<sup>10</sup>, Nahar<sup>11</sup>] - or laser based PIV as here - are useful for verification.

## EXPERIMENTAL SETUP

The U-tube arrangement, lower part, is shown in Fig.3. Two vertical pipes with inner diameter 4 cm and 8 cm respectively are connected via a bottom-assembly. In a drilling process this could correspond to the inner of a drill-string as one pipe and the other as the surrounding return annulus for the mud flow. The valves on each pipe can be closed individually to allow different flow and pressurization scenarios. The rig has deliberately been made more complex than an ordinary U-tube, in order to impose the effect of several unknown elements as in practical drilling.

### Instrumentation

Pressure transducers are positioned at the bottom and also in the zone where the gas bubble (gas pocket) can be placed under the valve in the left pipe (4 cm id).

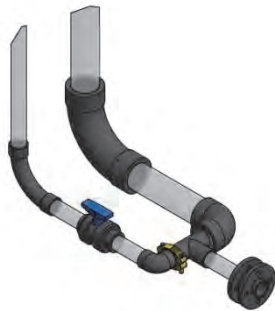
The pressure is logged using a PC with a National Instruments logging card with sampling frequency 250 Hz. The pressure sensors, both of type Rosemount 3051C, have a range of 620 mbar, and with a time resolution of 50 milliseconds (equals an update frequency of 20Hz), and a pressure resolution of 0.7mbar. A high speed camera (SpeedCam MiniVis-e2) was used to monitor the movement of the liquid interface. It was also used to calculate the velocity profile in the pipe using PIV.

The camera records up to 2500 fps at full resolution 512x512 pixels. It can record up to 120.000 fps at reduced resolution. The camera has onboard memory for 8223 full frames at full resolution. Images are downloaded to computer via a GigaBit Ethernet cable by means of a dedicated interfacing program ("*MotionBlitz*", by *Mikrotron company*).

For the PIV recordings a continuous wave (CW) diode laser (type "*Suwtech*") was used. It gives a beam with adjustable energy up to 200mW, and fixed wavelength (532nm).



a)



b)

Figure 3. Experimental setup showing the lower part of the U-tube. Both vertical pipes are fixed to a vertical aluminium bar, extending 3m above the valves. Liquid may be drained at lower left.  
 b) Drawing of bottom assembly (from bachelor thesis, E. Kvamme and K.H. Tjelta, 2013).

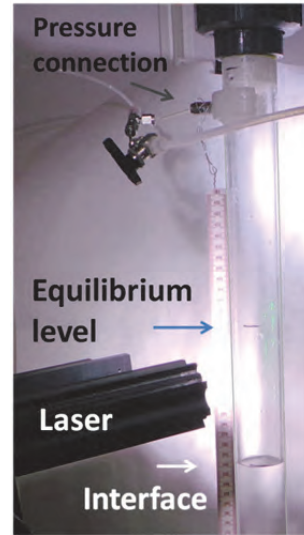


Figure 4. Picture of the bubble placed under the valve before start of oscillation.

The laser beam was expanded into a 1mm thick and 5 cm wide, nearly parallel “light sheet”. This was obtained using a cylinder expander lens, in sequence with a collimator lens. LED lamps were used in addition for the high-speed recordings when PIV images were not needed.

#### Fluids used for experiments

Both tap water and CMC (CarboxyMethyl Cellulose) was used. The behavior of the 1g/L solution is shown in Fig.5. A Carreau-Yasuda model is often used to describe the rheology of CMC. However, the measurements at low shear rates were not done in the tests here so a full parameterization was not feasible.

The elastic properties might in principle impact on the oscillatory motion in U-tube. The typical large scale flow oscillation frequency is of the order of seconds, and for finer flow details of the order of milliseconds. More detailed analysis of these properties will be discussed in the conference presentation of the paper.

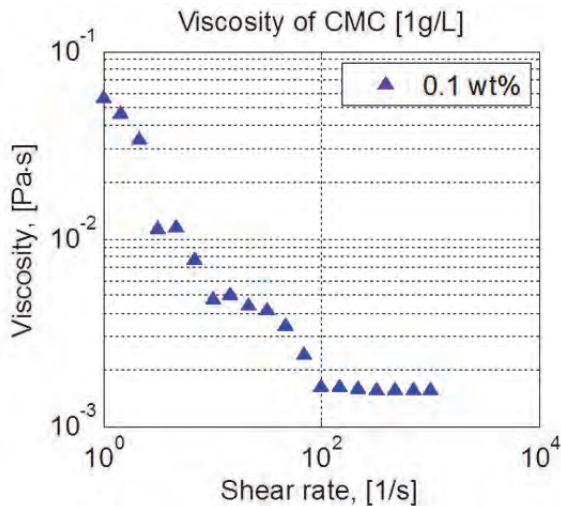


Figure 5. CMC rheology under flowing conditions.

### Experimental procedure

The gas bubble is set initially with the valve in the left pipe (i.d. 4 cm) closed and the valve in the right pipe open. The liquid level is nearly to the top of the pipes. The air bubble is supplied through the pipe inlet used for the pressure transducer (see Fig.4), exposed to the hydrostatic overpressure from the right pipe. When the bubble has been set, the gas supply is closed, and the right valve is also closed. Hence, the bubble still has the same overpressure as before the valve in the right is closed. Now a valve leading from the liquid outlet (lower left in the photo in Fig.3a ) is opened and liquid flows out due to the overpressure. The liquid is collected and then poured back on top of the right pipe, while still keeping the valves closed. Obviously the hydrostatic pressure over the right valve is higher than in the bubble. It is also straightforward to show by mass conservation of gas and liquid that when the right valve is opened, the system will return to exactly the same initial state, after thermal effects have returned to equilibrium. Heat is generated in the adiabatic compression and must be given time to dissipate. The return to the original state takes place in the same way as for an unforced damped harmonic oscillator,

except for initial hydrostatic unbalance. This is in close analogy with a spring and mass oscillating e.g. in a viscous medium.

All the experiments are carried out as described above. Pressure recording and high speed video are synchronized in a simple manner: one operator opens the valve in the right valve, at the same time as two other operators start the pressure recordings and the camera recordings. The whole oscillation experiment is usually finished within 15 to 20 seconds.

### Example run with tap water

In this example the gas bubble was 364 ml in the pressurized state. The two main valves were closed and the draining valve at the bottom was opened so that pressure was reduced to standard conditions, 1bar and 15 degrees Celsius. The gas bubble (pocket) for the present recordings then expanded to 540 ml.

In Fig.6 is shown the transient oscillation of pressure versus time. The pressure indicated is the gauge pressure relative the ambient atmospheric pressure. A Fast Fourier Transform (FFT) algorithm was applied to the pressure recordings in order to determine the oscillation frequency. The dominant frequency is 2.33 Hz which is the same as found by observing the high speed video recordings. Another peak may be seen at 20 Hz. This is connected to the update time of the pressure sensors. In the inserted magnified plot of the pressure this is clearly seen as a partly stepping pressure behavior. This should not be confused with the digital pressure resolution of the sensors (0.7mbar).

In Fig.7 instantaneous images of the gas-liquid interface are shown, as it passes the equilibrium level. These are to be compared with Fig. 9 which shows the velocity profile. When the interface passes the equilibrium level the pressure in the right and left pipes are equal. Hence in this situation there is no net driving pressure, or pressure gradient, only the hydrostatic variation up-down in the pipe.

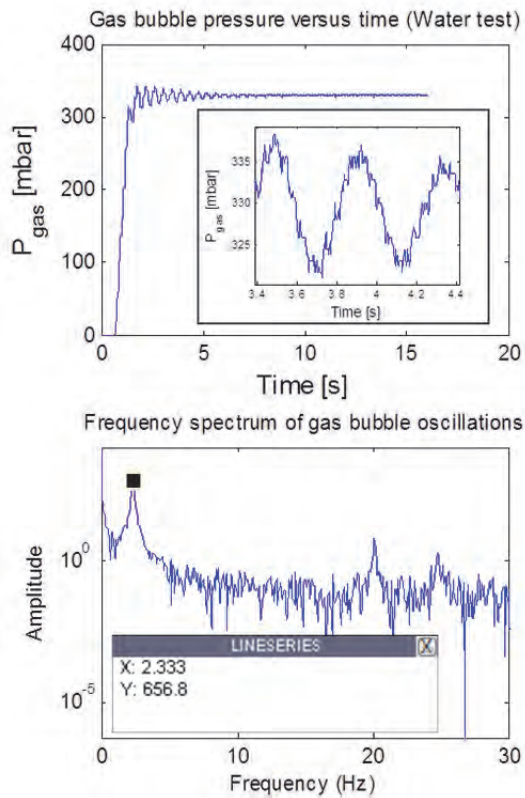


Figure 6. Top: Pressure versus time in the oscillating U-tube experiment with water. The FFT analysis is shown below.

This corresponds to the zero (crossover) points of the pressure gradient and it is seen that the flow profile is plug like. However as the drive pressure gradient is approaching minimum the flow will reverse. However it is seen that the velocity is zero in the middle of the pipe, but reverses closer to the pipe wall. This is easily seen in the video images Fig. 7b) and 7c). This leads to a peculiar situation where the centre region of liquid remains is at rest, while closer to the wall the liquid drains down. Thus a “stalagmite”-like structure stands frozen for several milliseconds before collapsing.

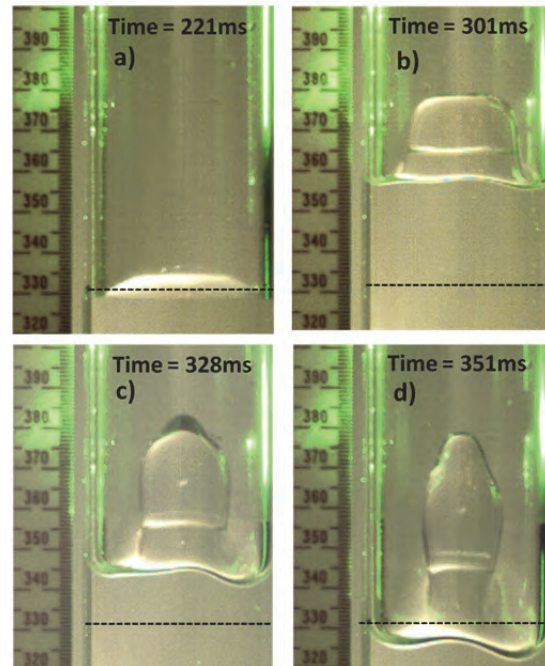


Figure 7. Images of the liquid interface during the first oscillation cycle. a) Liquid level just passing the equilibrium level upwards. b) Shortly after passing the equilibrium. c) Liquid stagnant at the top, while flow returns at the walls. d) Liquid interface crossing the equilibrium downwards.

### PIV analysis

Also PIV was used, to complement the pressure recording and video images. Tracer particles were added to the liquid to visualize the flow movement.

An open source Matlab program (MatPIV, Svein<sup>12</sup>) was used to perform two-dimensional cross correlation of the particle movement between two images taken at short intervals ( $\Delta t = 2$  ms). In Fig.9 is shown frame 799 with the calculated velocity profile inserted. The frame time is 1596 ms. This is 1126 ms after start, and corresponds to the third oscillation cycle, similar to those seen in Fig.6.

The PIV images (8800 images) are recorded with image centre at the initial interface position. This level is 13 cm below the equilibrium liquid level. Thus the PIV analysis can capture flow details slightly below the interface during the oscillations.

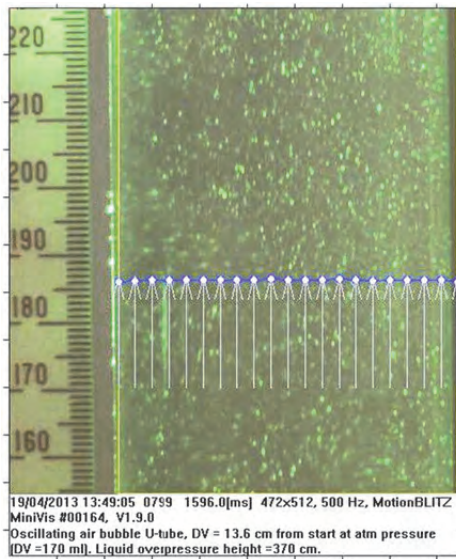


Figure 9. PIV image used for cross 2D correlation. Inserted are arrows showing the averaged velocity profile over the whole image cross section. This particular profile at frame 799 is 1126 ms (third cycle) after start. The velocity (0.17 m/s) is practically constant over the cross section.

### Example run with CMC

In the following example CMC was used instead of water. The experiment was repeated at the same conditions as for water. A quite low concentration of CMC (1g/L) was used to ensure good optical conditions for the PIV measurements, and to avoid small air bubbles to entrain the liquid for long time.

The pressure oscillations in this case are shown in Fig.10. It turns out that at least for this concentration of CMC – less than that used for drilling purposes – the oscillation frequency is almost the same as for water. The damping is also quite similar. Experiments with higher concentration will be carried out as a continuation of this work and paper, in order to determine the effects on oscillation frequency and damping.

## NUMERICAL SIMULATION

In order to compare the experiments with theory beyond the analytical solutions in equations (3) and (4) it was necessary to write a numerical simulator to include at least some of the many parameters that impact on the flow behavior. A Matlab program was written to include geometry of the flow rig as well as the time varying liquid level. It includes the gas pocket compression and fluid friction. The program is simulating a set of time dependent second order nonlinear equations.

It is based on a version of Newton’s second law and mass conservation for gas and liquid. Written in a highly symbolic way, it is analogous to the un-driven damped oscillator with equation of motion,

$$m\ddot{x} + c\dot{x} + mgx + kx = 0 \quad (5)$$

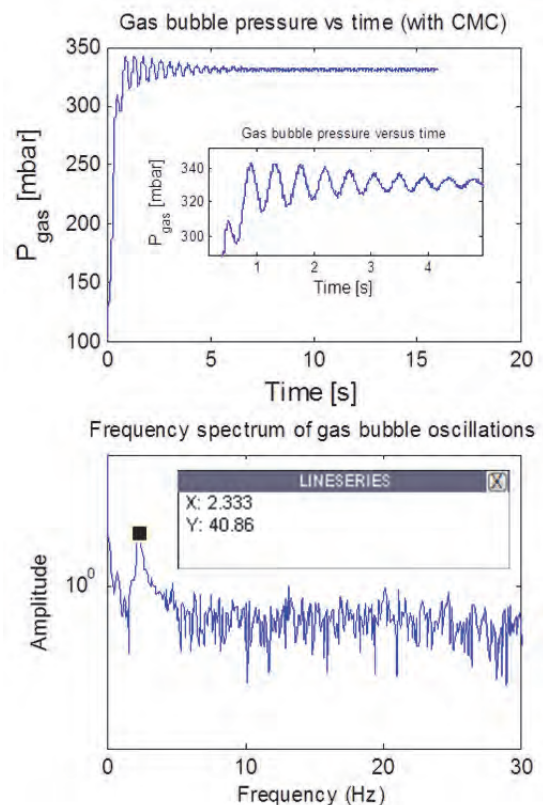


Figure 10. Pressure versus time in the gas bubble with CMC with FFT analysis below. Inserted at top the initial transient.

where  $m$  is liquid mass,  $x$  is displacement. The gas compression is described by using  $k$  as elasticity constant,  $c$  is connected to liquid viscosity, and “ $m'g'x$ ” represents the differential hydrostatic head for the oscillating column. The simulator includes the geometrical features of each section, the fluid properties and friction calculations. The compression of the gas bubble follows the equation of state for air, and has an option for simulating both isothermal and adiabatic compression.

There is no periodic external driving force except for the time varying hydrostatic pressure difference. The acceleration is connected to the fraction of total mass in the two parts with different pipe diameters. The damping factor is wall friction together with so-called “singular pressure drops” in bends, expansions and valves. The solution procedure is to split the movement from start to end into small time-steps (1ms intervals) and solve for kinematics and dynamics.

Initially the acceleration is calculated. This is essentially based on the net hydrostatic driving force, minus the friction forces. The net hydrostatic force is derived from the liquid level differences. Acceleration is calculated by dividing the sum of forces by the liquid mass. The acceleration and velocity in the two parts with different diameters are different, but are linked by the continuity equation. The speed and acceleration in the small left pipe is four times higher than in the right pipe, since the diameter ratio equals two. After the acceleration has been found, the velocity is calculated based on the velocity at the previous time-step. Finally the position of the interfaces is updated based on the position at the previous time-step with the additional movement due to velocity and acceleration in the new time-step.

All calculations are thus done in accordance with standard classical kinematics. Provided the time steps are small enough the error in assuming constant

acceleration is small. Since the time steps were so small, no special integration technique was needed, like e.g. Runge-Kutta or similar. The program itself is too big to be listed here, but will be shown in the conference presentation. The water experiment case described earlier was simulated. The gas pressure versus time equivalent to Fig.6 is shown in Fig.11 below.

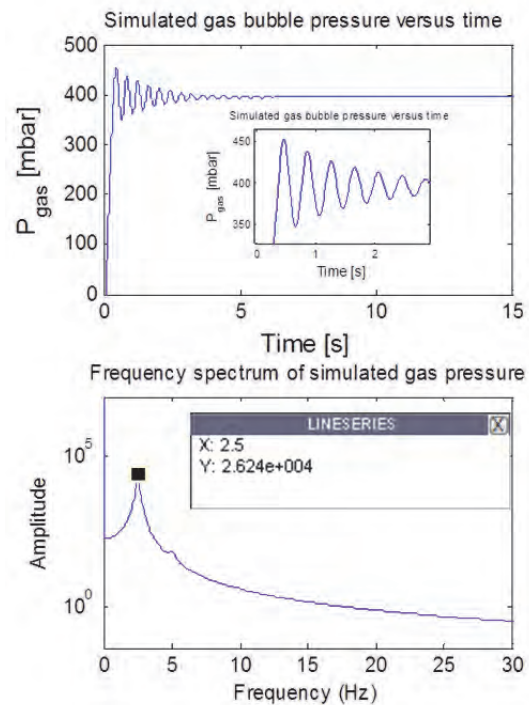


Figure 11. Numerical simulation of bubble pressure versus time as for the water test. The oscillation frequency becomes higher than found from the experiments.

The numerical simulations seem to overestimate the oscillation frequency slightly, at the same time as the initial damping is too low. For the classical damped harmonic oscillator it is well known that the friction term also impacts on the oscillation frequency. The friction has been calculated using the standard pipe flow formulas for steady laminar and turbulent flow. However, all the experiments show similar results, that the frictional damping is much higher than predicted by simulation. In fact during the initial transient the



turbulent friction factor in the simulations for the two first seconds had to be multiplied with a factor 15 in order to damp the initial oscillation to a level which was comparable to the experiments. However there have been discussions, as in Manero<sup>18</sup>, that oscillatory motion of non-Newtonian liquids might increase flow rate for certain pressure gradients

## DISCUSSION

It is seen from the numerical simulations for water that although the “correct” friction factors are used, the damping is not properly described unless a considerably higher friction is used. This shows that for oscillatory flows the standard way of calculating pressure drop – based on steady state flow – is not correct for unsteady flows. This applies for Newtonian as well as non-Newtonian flows.

The effective friction for CMC could have been calculated using e.g. Metzners<sup>14</sup> or Pilehvari<sup>15</sup> models. But in view of introducing even more internal and often unknown parameters the simplest models were used here.

The use of high-speed images and movies reveals that the movement of the liquid interface during parts of the oscillation sequence is complex. The interface is strongly impacted by inertia as it approaches the highest level. It was also found that the flow below the liquid interface was not radial symmetric. This is geometry induced; the flow is stronger on the rear side of the pipe. This is most likely caused by the bend at the bottom of the small pipe. It creates an asymmetric flow profile, stronger at the “outer” side of the turn. This was reflected as a stronger uplift of the interface at the rear side. From friction point of view it also enhances shear stress on that side.

### Effect of liquid compressibility

It is a relevant question to ask if the CMC would respond differently under

compression than water would do. However, at least with the low concentrations of CMC used here, the elastic module found from the rheometric tests were too small to compete with the overall uncertainties of effects in the pipe friction. Most likely it would contribute more in a deep well with high liquid overload to accelerate during oscillations.

### Effects of fluid flow and pipe geometry

In the present experiments and in the numerical simulations it is found that the flow during oscillations is slightly turbulent only in the first initial transient. The velocity is at maximum around 1m/s. Later in the sequence - as is also seen from the PIV images in Fig.9 - the flow straightens and becomes laminar.

For real drilling operations local uneven geometries, might induce flow instabilities. Also the effect of eccentricity in annulus systems is strong for non-Newtonian fluid flow, and may lead to substantial variation in pressure drop (Pilehvari<sup>15</sup>). Also the flow during drilling can be close to turbulent, causing the picture to become even more complicated from a rheological point of view.

As can be seen from a comparison of the experiments and the numerical model the dynamics is much more complex than predicted from only hydrostatic forces and friction. It is possible that the acceleration term going from one diameter to another could be important. There are semi-theoretical techniques available, but this has not been addressed in the simulations. Another interesting option would be to do an investigation by PIV.

## CONCLUSIONS

The aim of this work and paper was to investigate how rheological parameters enter large scale flows in pipes and even more for complex well drilling.

An experimental study of medium scale U-tube flow oscillations has been carried

out. It has involved several types of flow detail analysis, in order to be able to address uncertainties in system behavior. The impact on oscillations in both Newtonian and non-Newtonian flow was demonstrated. It turns out that the effect of rheological parameters on the overall flow behavior in complex pipes and wells is not straightforward.

Overall flow dynamics and friction aspects are not easily determined even though the rheological parameters are accurately determined. The shear thinning properties of course contribute to reduce flow friction against the pipe walls, but for drilling operations the boundary conditions are not known down-hole. This means that also the shear conditions and effective shear stress are less predictable.

Although in real drilling operations of wells the pressure drop is estimated as accurately as possible, such operations are not scientific experiment. They involve costly technologies, and whatever resources put into the work it is virtually impossible to know the down-hole geometry accurately enough. When predictions and calculations of pressure drop do not fit “reality”, this is usually explained with eccentricity of the drill-string and to the “pipe roughness”.

One very important part of knowing the rheology of the drilling fluid remains. This is the “local behavior”. One of the most important jobs of the drilling fluid is to be able to transport the drilled particles – “cuttings” – to the surface. The shear thinning behavior, “thickening” towards low shear rates, takes care of this. However, even if this is a more “local effect” it turns out that even cuttings transportation and “hole cleaning” is a similarly difficult, and also another story.

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