Small and Large Amplitude Oscillatory Shear Response in Non-Isothermal Flow of a PTT Fluid in Lid-Driven Polar Cavity

Hatice Mercan, Kunt Atalık

Mechanical Eng. Dep., Boğaziçi University, 34342 Bebek, Istanbul, Turkey

ABSTRACT

Oscillatory response of non- isothermal viscoelastic PTT fluid flow is studied in a polar cavity with a moving lid. The effects of frequency and amplitude of the excitation on the stress and thermal fields are demonstrated and compared in terms of isotherms, Lissajous curves and frequency analysis based on FFT.

INTRODUCTION

In this study we analyze the oscillatory non-isothermal viscoelastic flow in a polar cavity. The oscillatory excitation is induced by the motion of the moving inner curved lid. The cavity geometry with boundary conditions is shown in Fig.1 (a). Lid Driven cavity flows with oscillating lid are widely used to understand the mixing properties of the flow fields and heat and mass transfer enhancements. Most of the studies present in the literature are for square cavities and Newtonian fluids. Ghasemi and Aminossadati[1] presented a numerical investigation of unsteady mixed convection heat transfer in lid-driven square cavity comparing the constant and oscillatory moving lid cases for a Newtonian fluid. The study revealed the heat transfer rate changes due to oscillatory motion of the lid compared to constant lid case. At high buoyant forces, the heat transfer rate for constant lid case is found to be higher compared to the oscillatory lid motion case. Nishimura and Kunitgusu[2] numerically investigated fluid mixing and mass transfer in cavities with oscillatory lid motion. They revealed that there is an optimum frequency of the lid velocity for the best mixing and that excellent global fluid mixing does not always lead to enhancement of heat and mass transfer. Nishimura et al.[3] presented an experimental study and described mass transfer enhancement with oscillation frequency in grooved channels with different cavity lengths for pulsative flow. Khanafer et al.[4] carried out a numerical investigation of unsteady laminar mixed convection heat transfer in lid-driven cavity under a range of inertial, buoyant forces with oscillatory conditions for a Newtonian fluid. Karimipour et al.[5]numerically investigated the periodic mixed convection of a water-copper nanofluid inside a rectangular cavity. They presented the streamlines and isotherms in a period for different volume fractions of nanofluid.

The number of studies present in the literature about viscoelastic oscillatory investigation of the lid-driven cavity flow is rather few. Most of the oscillatory response investigations are done for the simple shear flows between two plates in order to understand the rheological properties of an a priori unknown substance like polymer melts, suspensions, biological fluids etc. Experiments and numerical simulations are carried out to understand the material responses under small and large amplitude oscillatory shear excitation[6]. The non-linear effects become more distinctive with increased amplitude of the perturbation in oscillatory shear flow and they are generally investigated by means of the Fourier analysis. Wilhelm[7] presented the Fourier spectra of experimental data of shear thinning polymer (polypropylene) and revealed the stress harmonics. Through experimental and numerical studies Ewoldt et al.[8] investigated large amplitude oscillatory shear response of pseudo-plastic and elasto-visco-plastic materials. They introduced a new scalar parameter, the perfect plastic dissipation ratio, which uniquely identified the plastic behaviour of the material. Fetacua and Hayat[9] carried out an analytical study to establish exact solutions for some oscillating motions of Oldroyd-B fluid. They concluded that for sine and cosine oscillations, the time required to reach steady state decreased with the reduction of the frequencies of the boundary velocity. Nam et al.[10] investigated the phase angle (δ_2) of the first normal stress difference in oscillatory shear flow using both experimental and simulation results. In order to reveal the nonlinear behaviour of the first normal stress difference (N_1) in large amplitude oscillatory shear they introduced the magnitude and phase angle of the forth harmonic to the nonlinear response.

The flow in periodically driven cavities has applications in polymer processing and micro-fluidics. For this reason the oscillatory response of the fluid inside a cavity has been investigated experimentally and numerically under low concentration, low elasticity and high inertia. Sriram et al.[11] performed the experiments with polyacrylamide-water solution using Power-Law model in the numerical simulations. However, there were discrepancies between the experimental and numerical results where streamline patterns diverged with increasing elastic effects (Weissenberg number/Reynolds number ratio).

In this study we investigate the thermal and elastic effects for a PTT fluid under the sinusoidal motion of the moving lid in a polar cavity. The material properties (such as polymer viscosity and relaxation time) are considered temperature dependent and general Williams-Lendel-Ferry model is used to describe the temperature dependency.

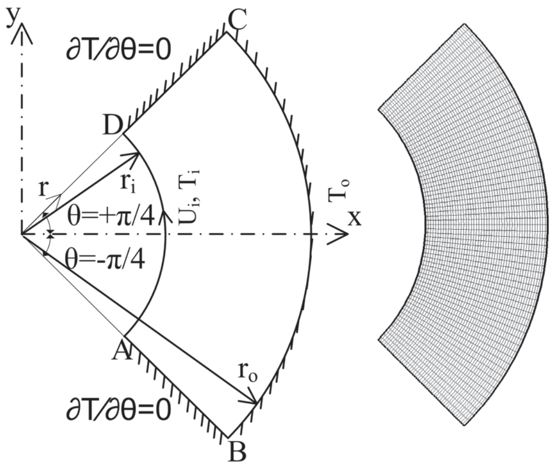
The phase diagrams, frequency spectrums and isotherms of non-isothermal PTT fluid under oscillatory motion for low and high frequency and amplitude couples are shown.

FORMULATION AND METHOD OF SOLUTION

The two dimensional flow of a polymer solution in a polar cavity with an oscillating lid is considered. The cavity angle is fixed to $\theta_p=\pi/2$. The radius ratio is (ratio of inner radius r_i to outer radius r_o) fixed to $r_i/r_o=1/1.75$ (see Fig. 1 (a)). The inner curved wall moves in θ direction with a sinusoidal velocity distribution. The no-slip boundary conditions are specified at the walls. The inner curved wall velocity is given by,

$$U_i(t,\theta) = \frac{8}{(\pi/2)^4}\left[1+\tanh 8t-4\right] \left(\frac{\pi}{4}-\theta\right)^2\left(\frac{\pi}{4}+\theta\right)^2 A\sin(2\pi\varpi t) \quad (1)$$

where t denotes the time, A is the non-dimensional amplitude and ϖ is the frequency of the excitation. The validation of the distributed velocity definition can be found in a previous study of the authors[11]. The outer wall temperature T_o is higher while the temperature of the moving inner curved wall T_i is lower ($T_o>T_i$). The side walls are assumed to be adiabatic.



(a) (b)

Figure 1. (a)Cavity Geometry[12], (b) Sample Grid (65×49).

The mass and momentum balance equations for incompressible, non-isothermal, viscoelastic flow are given in dimensionless form as,

$$\nabla \cdot \underline{V} = 0 \tag{2}$$

$$\mathrm{Re}\frac{D\underline{V}}{Dt} = -\nabla p + \beta(T)\nabla^2 \underline{V} + (1-\beta(T))\nabla \cdot \underline{\underline{T}} \tag{3}$$

where $\underline{V}$ is the velocity vector, p is pressure, $\underline{\underline{T}}$ is the viscoelastic extra-stress tensor. T is temperature and D/Dt denotes the material derivative. The Reynolds number is defined as $\mathrm{Re} = \rho V_0 r_i / \mu$, where ρ is the fluid density, V_0 is the characteristic velocity and μ is the total shear rate viscosity. The range of Reynolds number is $0.3 \le \mathrm{Re} \le 1$. The dimensionless parameter β is the ratio of the solvent viscosity (μ_s) to the total zero-shear rate viscosity ($\mu = \mu_s + \mu_p$, with μ_p, the polymer viscosity).

In this study Phan-Thien-Tanner constitutive relation is used to model the viscoelastic fluid. In dimensionless form the Phan-Thien-Tanner model reads,

$$\underline{\underline{T}} + We(T)(\overset{\nabla}{\underline{\underline{T}}} + \varepsilon tr(\underline{\underline{T}})\underline{\underline{T}}) = 2\underline{\underline{D}} \tag{4}$$

where $\underline{\underline{D}}$ is the rate of deformation tensor, ε is the elongational parameter of the PTT model. This parameter characterizes the strain softening/hardening and shear thinning feature of the polymer additive. Weissenberg number is defined as $We = \lambda V_o / r_i$, where λ is the relaxation time of the fluid. The operator $\overset{\nabla}{(\ )}$ denotes the upper convected derivative as follows,

$$\overset{\nabla}{\underline{\underline{T}}} = \frac{D\underline{\underline{T}}}{Dt} - \underline{\underline{L}}^T \cdot \underline{\underline{T}} + \underline{\underline{T}} \cdot \underline{\underline{L}} \tag{5}$$

where $\underline{\underline{L}}$ is the velocity gradient. The energy equation reads

$$\frac{DT}{Dt} = \frac{1}{Pe}(\nabla^2 T + Br(\underline{\underline{T}} : \underline{\underline{D}})) \tag{6}$$

with Péclet number defined as $Pe = \rho C_p V_0 r_i / k$, where C_p is the heat capacity and k is the thermal conductivity, and it has been fixed as Pe=900. Brinkman is defined as $Br = \frac{\mu V_0^2}{k(T_w - T_0)}$ and it characterizes the viscous dissipation. In this study the value of Brinkman number is in the range of $5 \le Br \le 50$. The non-dimensional temperature is defined as $T = \frac{\tilde{T} - T_i}{T_o - T_i}$ where $\tilde{T}$ is the dimensional temperature. In this study the density and thermal conductivity of the fluid are assumed independent of temperature field. However polymer viscosity μ_p and relaxation time of the polymer additive λ are assumed to be temperature dependent. The temperature dependence of the non-dimensional Weissenberg number and viscosity ratio are as follows

$$We(T) = Wef(T)$$

$$\beta(T) = 1 - \omega_r f(T) \qquad (6)$$

where ω_r is the retardation parameter defined as the ratio of $\frac{\mu_p}{\mu}$ and the temperature dependency function, *f(T),* is defined according to Williams-Lendel-Ferry (WLF) model which reads;

$$f(T) = \exp\left[-\frac{c_1 T}{c_2(T_o - T_i) + T}\right] \qquad (7)$$

We set the quantities $T_o - T_i = 1$, c_1=15 and c_2=50[12].

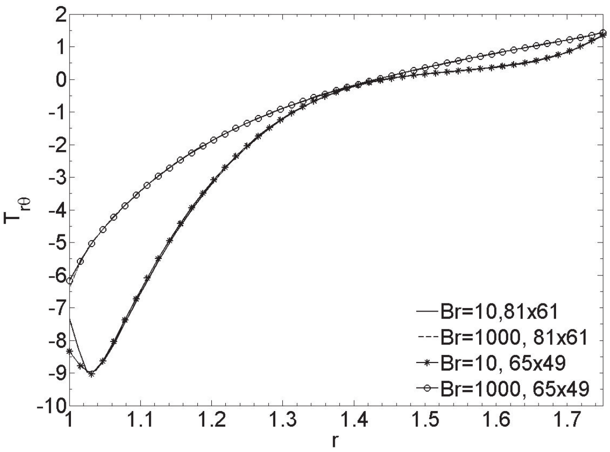


Figure 2. Comparison of shear stress distributions along radial direction (at θ=0.2945 rad.) for non-isothermal viscoelastic constant lid velocity case for two grid densities (81x61 and 65x49) and Brinkman values (Br=10, Br=1000) at Re=0.5, We=1.0, ω_r=0.4, ε=0.3, Pe=900.

The governing equations are solved numerically in cylindrical coordinates with stream function-vorticity formulation. The explicit Runge-Kutta-Fehlberg formulation is used for time integration. In this study the steady state solutions for the case of a lid moving with a non-oscillatory velocity are used as initial conditions for the oscillating lid case. The governing equations are discretized in space with a second order centered finite difference scheme. The elliptic equation is solved by successive over relaxation (SOR) method with Chebychev acceleration. The grid independency test results are shown in Fig. 2 and Table 1 for different grid densities. A structured grid with (65x49) grid density is chosen for simulations (see Fig. 1(b)).

Table 1. Minimum stream function values/locations and maximum stream function values for three different grid densities at Re=0.5, We=0.5, ω_r=0.3, ε=1.0, Pe=900 and Br=10.

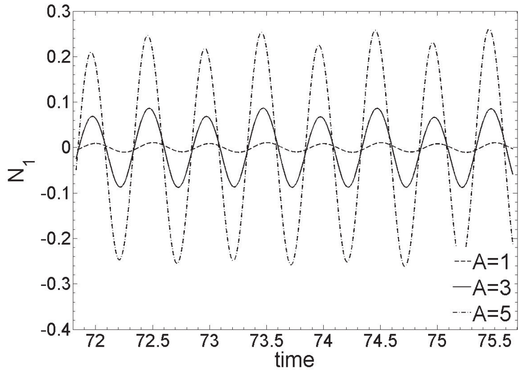
Grid	Ψ_{min}	(x_{min},y_{min})	Ψ_{max}
51×39	-0.0948415	(1.2342,-6.42e-5)	9.11e-6
65×49	-0.0948958	(1.2260,-6.75e-5)	6.86e-6
81×61	-0.0948845	(1.2250,-7.28e-5)	5.07e-6

RESULTS AND DISCUSSION

In this study the non-isothermal viscoelastic PTT fluid flow is examined under oscillatory excitation. The stress and thermal responses are shown for different frequencies and amplitudes.

The effect of excitation amplitude at low inertia in non-isothermal viscoelastic flow:

We first investigate the effects of amplitude to first normal stress difference (N_1) and the average Nusselt modulus at the oscillating inner lid. The Nusselt modulus used in this study is defined as $Nu = r_i h / k$, where *h* is the heat transfer coefficient at the wall, and it is calculated from $q_w = h(T_0 - T_i)$ and *k* is the thermal conductivity. In Figs. 3(a) and 3(b) the time evolution of first normal stress difference and Nusselt modulus are shown, respectively, for non-isothermal viscoelastic PTT flow at Re=0.3, We=1, ω_r=0.55, Pe=900, Br=5, ϖ=1.0 and ε=1, for increasing amplitudes. It is observed that the maximum values of the stress and temperature responses increase with increasing excitation amplitude (Fig. 3).



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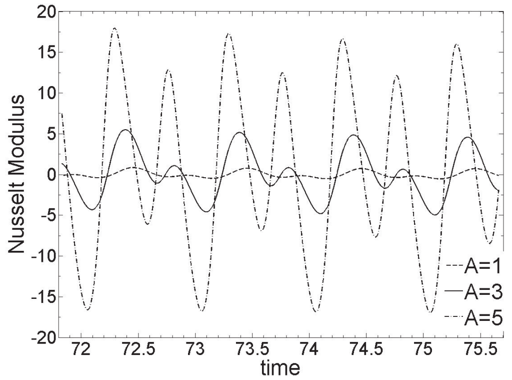
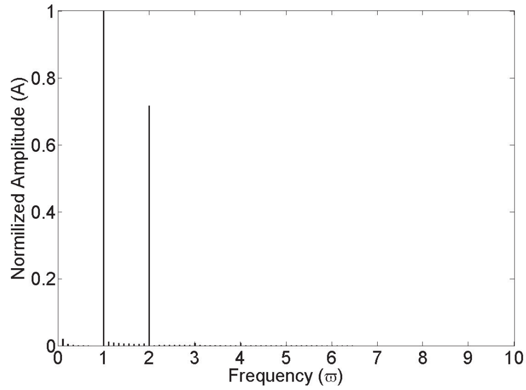


(b)

Figure 3. Time evolutions for increasing amplitudes at Re=0.3, We=1, ω_r=0.55, Pe=900, Br=5, ϖ =1.0 and ε=1 (a) First normal stress difference vs. time (b) Nusselt modulus vs. time.



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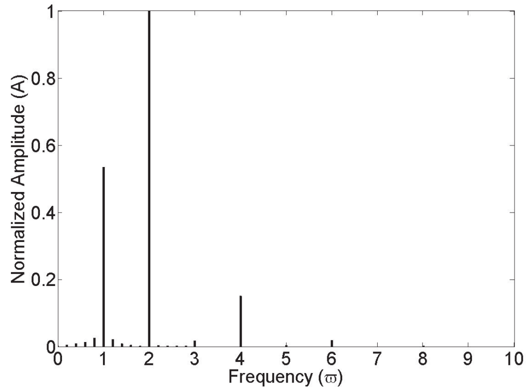


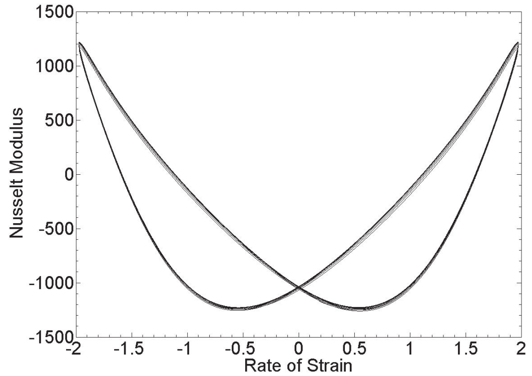
(b)

Figure.4 Fourier spectrum, Re=0.3, We=1, ω_r=0.55, Pe=900, Br=5, ϖ =1.0 and ε=1 (a) A=1, (b) A=5.

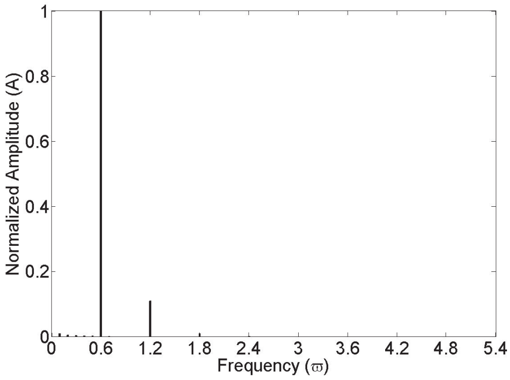
The result of the frequency analysis is shown in Fig. 4(a) and (b) for Nusselt modulus at amplitude A=1 and A=5, respectively. At low amplitudes the excitation frequency ϖ =1.0 and an even harmonic are observed. However as the amplitude increase the even harmonics of the excitation frequency are more pronounced (Fig. 4(b)).

<u>The effect of oscillation to temperature and stress fields and Nusselt modulus:</u>

In this case the effect of oscillation of the moving lid on temperature field is investigated. The phase diagram and the frequency spectrum are shown in Fig. 6 for viscoelastic PTT case Re=1, We=1, ω_r=0.55, Pe=900, Br=50, ϖ =0.3, A=5.0 and ε=1. The even harmonics in the frequency spectrum are observed in the Figure 6(b). The time evolutions of the isotherms are shown in Fig. 7 for one period after the steady periodic state is achieved. The snapshots points for isotherms are shown in Figure 8. It can be observed that the temperature gradient is greater near

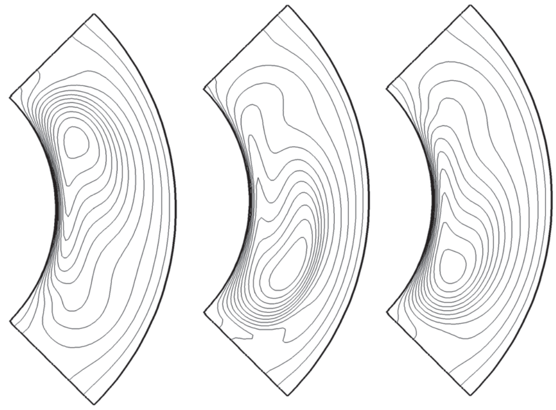


(a)

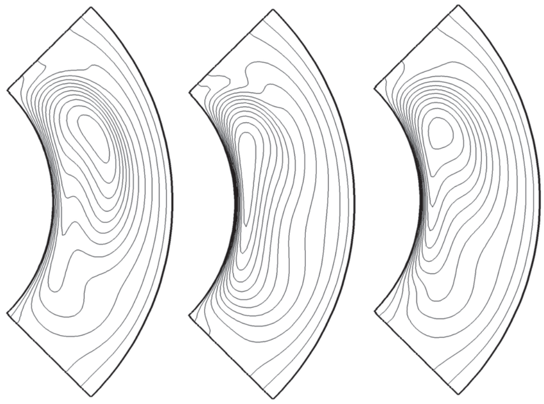


(b)

Figure 6. Re=1, We=1, ω_r=0.55, Pe=900, Br=50, ϖ =0.3, ε=1 and A=5 (a) Lissajous of Nusselt modulus (b) Frequency spectrum.



(a) (b) (c)



(d) (e) (f)

Figure 7. Isotherms in a period for Re=1, We=1, ω_r=0.55, Pe=900, Br=50, ϖ =0.3, ε=1 and A=5.

the oscillating lid and the peak value moves with the central peak of the isotherms. In Fig. 9(a) the Lissajous curve of the first normal stress difference is shown at two different frequencies for Re=1, We=1, ω_r=0.55, Pe=900, Br=50, ε=1 and A=5. The phase angle, δ_2, decreases with increasing frequency. ($\delta_2(\varpi$ =0.3)~80º, $\delta_2(\varpi$ =2)~40º) This is also indicated in the experimental study of Nam et al.[10] In Fig. 9(b) first normal stress difference versus rate of strain square is shown. The departure from ellipsoid confirms the nonlinear nature of the response[10]. In Fig. 10(a) the Lissajous curve of the Nusselt modulus is shown. The phase angle, δ_N, increases with increasing frequency. ($\delta_N(\varpi$ =0.3)~-55º, $\delta_N(\varpi$ =2)~80º). In Fig. 10(b) Nusselt

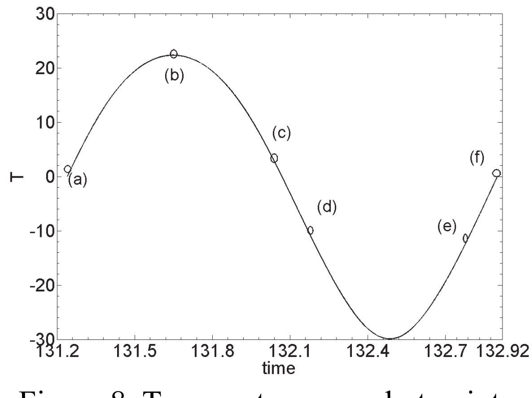
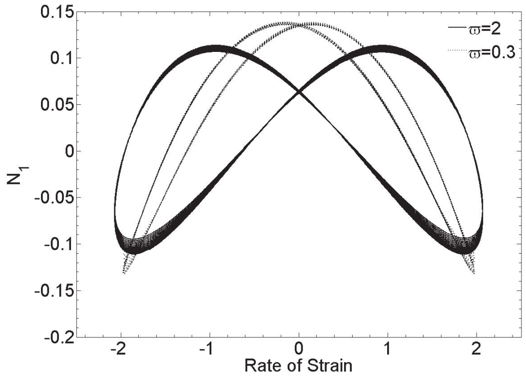
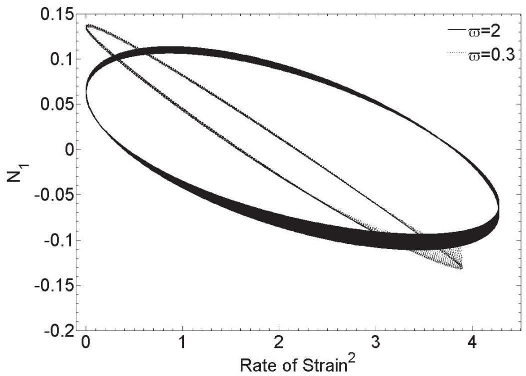


Figure 8. Temperature snapshot points.

modulus versus rate of strain square is shown. The departure from ellipsoid can be observed for both frequencies with excitation amplitude A=5.

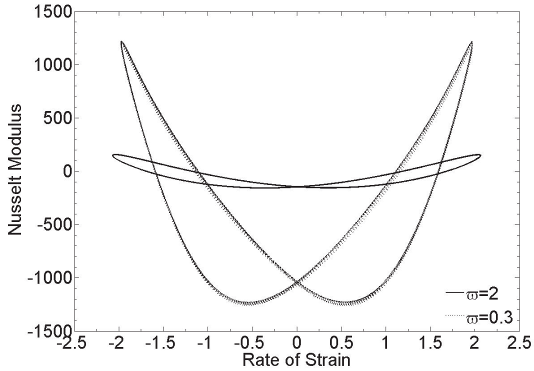


(a)

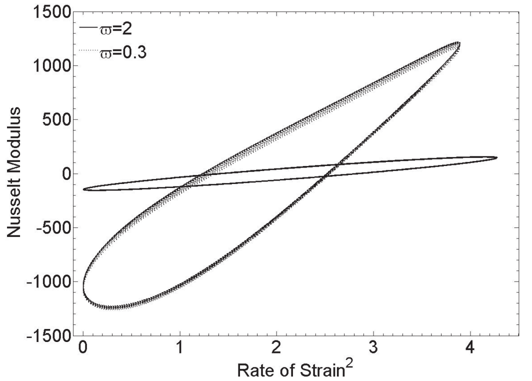


(b)

Figure 9. The first normal stress difference (N1) phase diagrams for Re=1, We=1, ω_r=0.55, Pe=900, Br=50, ε=1 and *A*=5 at two different frequencies (a) N1 vs. rate of strain (classic viscous Lissajous curve) (b) N1 vs. rate of strain square.



(a)



(b)

Figure 10. The Nusselt modulus phase diagrams for Re=1, We=1, ω_r=0.55, Pe=900, Br=50, ε=1 and *A*=5 at two different frequencies (a) Nusselt modulus vs. rate of strain (b) Nusselt modulus vs. rate of strain square.

CONCLUSION

In this study we investigated numerically the oscillatory response of a non-isothermal viscoelastic PTT fluid in a polar cavity. We first investigated the effect of amplitude to responses of the normal stress difference and Nusselt modulus, for moderate viscous heating effect and low inertia. Even harmonics of the excitation frequency are observed in the frequency spectrum of Nusselt modulus, their number and intensities are increasing with the excitation amplitude.

As the viscous heating and inertia effects increase the even harmonics are observed more clearly for the Nusselt modulus frequency spectrum (Fig. 6(b)). A non-linear response of the first normal stress difference and Nusselt modulus are observed with *A*=5 for low and high excitation frequencies.

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