Simulation of extensional flow through contractions towards a measuring system for extensional viscosity

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ABSTRACT

The flow through different contractions was evaluated by numerical simulations of Boger fluids, specially aimed at extensional flow. The simulations showed that a hyperbolic configuration was superior to the other geometries in achieving a constant extension rate throughout the nozzle.

INTRODUCTION

There are currently few commercial measuring systems available for measuring extensional rheology on medium-viscosity fluids, such as dispersions, foods and other biological systems. But, the interest in how a material behaves under tension has increased over the years as it has been discovered that many process-related problems occur due to elongation.

A motivation for determining extensional properties is that the extensional response, described by the extensional viscosity can differ considerably from the shear response. Rheological studies for the control of product properties of i.e. food products, polymers, dispersions or liquid medicine are thus central, enabling an assesment of the total quality of a product throughout the whole chain, from raw material to finshed product. It is also an important tool for process optimization and the ability to customize new product features such as consistency, transparency and sustainability.

The flow arising in contraction flows is a complex mixture of both shear and extension, with regions of strong shear along the wall and regions with strong extension at the centre of the flow.

Contraction flow through a hyperbolic nozzle, the Hyperbolic Contraction Flow method (HCF) has been suggested to be a suitable system for measuring extensional properties for fluids such as foods (e.g. dough), dispersions and medical systems by Wikström and Bohlin [1] and Stading and Bohlin [2-3] based on Bindings theoretical work [4]. The aim in the following work was to validate the assumptions made in the work by Wikström, Bohlin and Stading [2-3,5] . The aim was also to numerically evaluate the flow behaviour through the hyperbolic contraction and similar geometrical configurations to find an optimal geometry for achieving a constant uniaxial extensional flow. This would facilitate the extraction of extensional properties in a measuring system suitable for fluids in the medium viscosity range. Thus, the flow of elastic fluids with constant shear viscosity (i.e. Boger fluids) through several different axisymmetric contraction configurations was analysed through the strain, extensional viscosity and N_1 profiles. Here the results for the hyperbolic contraction and 45 degree contraction are presented, with a contraction ratio of 4:1.

GOVERNING EQUATIONS

An isothermal, incompressible viscoelastic fluid in motion is described by the constitutive equations of continuity and motion. They are expressed as follows:

$$\nabla \cdot u = 0 \qquad (1)$$

$$Re\left(\frac{\partial u}{\partial t} + u \cdot \nabla u\right) = -\nabla p + \nabla \cdot (\tau + 2\beta D) \qquad (2)$$

Here, u is the velocity vector, t the time, p the hydrodynamic pressure, D the rate-of-deformation $(\nabla u + \nabla u^T)/2$ and τ the total extra stress tensor. ß is the solvent-viscosity ratio parameter $ß = \eta_s/(\eta_s + \eta_p)$, η_s is the solvent viscosity and η_p is the polymeric viscosity. The non-dimensional Re-number is defined as: $Re = \frac{\rho U_{avg} L}{\eta_s + \eta_p}$, where ρ, L and U_{avg} are the density, the contraction gap width and the characteristic velocity (average upstream velocity) respectively.

A non-dimensional Weissenberg number, which depends on the characteristic time of the fluid (λ), density (ρ), characteristic velocity scale (U) and length scale (ℓ) is defined as: $We = \lambda U/\ell$.

NUMERICAL METHOD

In this study, numerical simulations of viscoelastic Boger fluids through different contractions were considered. Boger fluids were modelled by the mathematical model named the FENE-CR model, which display a constant shear viscosity and an almost quadratic first normal stress difference (N_1). The L-parameter was set to 5 and the We-number was varied in the range from We=0.1 to We=50. The numerical scheme used in this study to gain an approximate solution to the governing equations (1) and (2) is based on a combination of numerical strategies. It is a combination of the finite element and the finite volume method (fe/fv(sc)), based on a Taylor-Galerkin[6] and a pressure-correction scheme[7-8].

The flow domain of the two axisymmetric geometries used for the numerical analysis is discretised into smaller elements forming a mesh presented in Fig. 1.

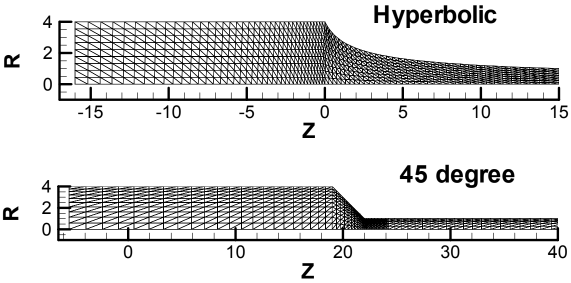

Figure 1. The triangular mesh used to model the fluid domain of the hyperbolic geometry.

The finite element mesh was made up of 1350 triangular elements, 2869 nodes and 17 974 degrees of freedom. Each finite element was divided into four finite-volume subcells forming a finite volume mesh for the stress (τ). On the vertices and mid-side nodes quadratic velocities and stress functions are located while linear pressure functions are located only at the vertices. The calculation procedure is performed by first calculating a divergence-free velocity field through a predicting and then a correcting step. This is followed by a pressure correction at a second stage and a third stage, where a new divergence free velocity field is calculated from the previous stages [7].

Due to symmetry, only half of the flow domain is used for the simulations. No-slip boundary conditions were imposed on the wall, a pressure-driven Poiseuille flow was prescribed at the inlet and a natural streamwise (open) boundary condition was imposed at the exit. The pressure at the exit was set to be zero and all solutions were studied under steady-state conditions.

RESULTS

Strain rate

The strain rate along the symmetry line (z) for the hyperbolic configuration was studied at different Weissenberg numbers. The numerical prediction for a Boger fluid flow is displayed through the 45 degree contraction in Fig 2 and through the hyperbolic contraction in Fig 3.

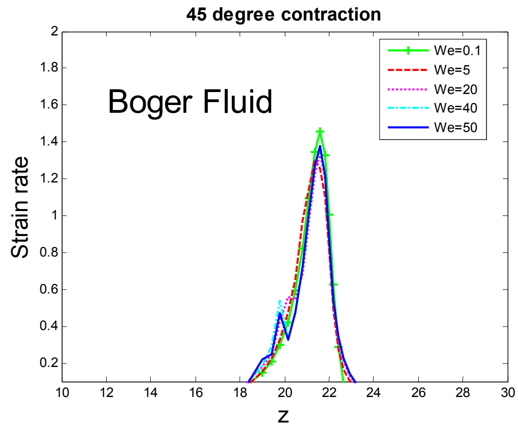

Figure 2. Strain rate ($dU_z dz$) along the symmetry line for viscoelastic flow at different We-numbers.

The strain rate in the 45 degree contraction is centred on the start of the contraction (*z=20*), where a peak reaching 1.46 units for We=0.1 is found.

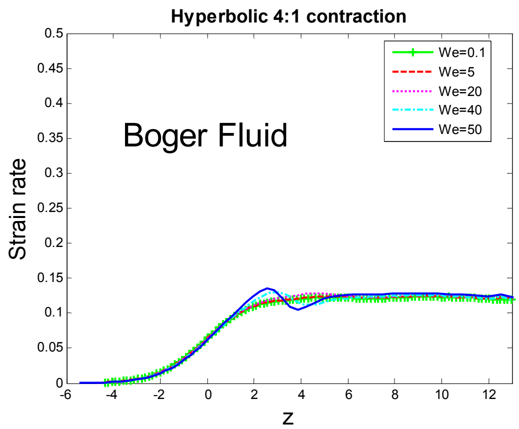

Figure 3. Strain rate ($dU_z dz$) along the symmetry line for viscoelastic flow at different We-numbers.

The strain rate of the fluid in Fig. 3 increases just before the start of the contraction (*z=0*) and reaches a constant level at around 0.12 units at z=5, until the end of the contraction. For We=50 it takes a longer time before the flow is stabilized compared to the lower We-numbers.

The First Normal Stress Difference

The first normal stress difference (N_1) is defined as:

$$N_1 = \sigma_{zz} - \sigma_{rr} \tag{3}$$

In Fig. 3 N_1 is plotted along the symmetry line at different Weissenberg numbers for the hyperbolic contraction.

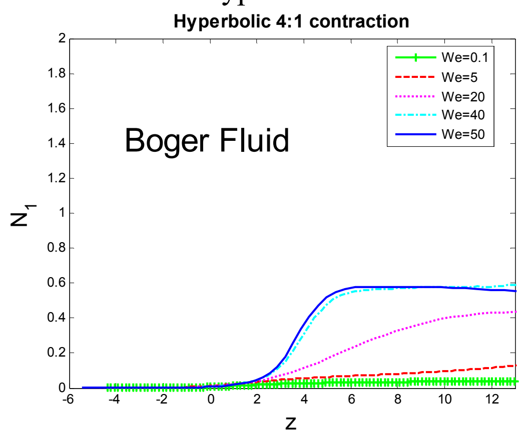

Figure 3. Normal stress difference (N_1) along the symmetry line at different We-numbers for viscoelastic flow.

The level of N_1 increases as expected for higher We-numbers reaching a level of 0.59 units for We=50.

Extensional viscosity

From the results of strain rate and N_1 an extensional viscosity can be calculated through the following equation:

$$\eta_e = N_1 / \dot{\varepsilon} \tag{4}$$

A plot of the extensional viscosity at the symmetry line for different We-numbers can be seen in Fig 4.

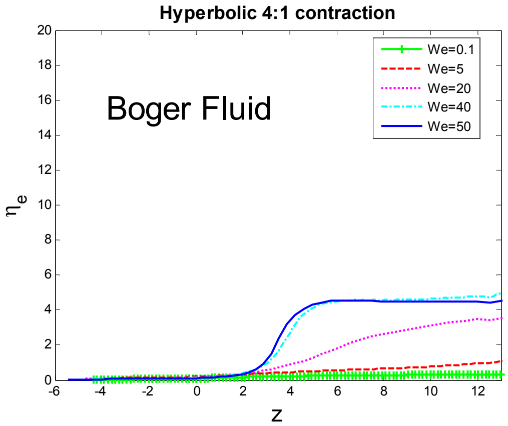

Figure 4. Extensional viscosity () along the symmetry line for a viscoelastic fluid at different We-numbers.

Naturally, the behaviour of the extensional viscosity through the geometry follows the behaviour of N_1. The increase in extensional viscosity starts after the start of the contraction and small increase is observed towards the end. For We=40, 50 a constant plateau with an extensional viscosity level of 5 units.

DISCUSSION

The results presented here show that a constant strain rate is achieved in the hyperbolic configuration in contrast to the 45 degree contraction, where a peak around the start of the contraction (*z=20*) is observed. The strain rate reaches a lower maximum value in the hyperbolic contraction but will undergo extension for a longer time period. This will in turn facilitate practical determination of the extensional viscosity of the sample.

Both the strain rate and N_1 are studied at different We-numbers. An increasing Weissenberg number is a measure of either increasing flow rate or increasing fluid elasticity. As the elasticity of the fluid is increased, an increase in N_1 is observed and a constant level of N_1 is sooner reached. Due to the gentle contraction of the hyperbolic configuration, lower forces are needed to push the fluid through the die. Higher forces can thus be used which will lead to a broader measuring range, enabling higher strains and strain rates. This also indicates that the use of a hyperbolic contraction in pipe contractions will generate a more uniform flow.

CONCLUSION

Various numerical predictions were performed on the flow of two different contraction configurations, a hyperbolic and a 45 degree contraction. All the simulations were modelled under steady-state conditions and a Poisson flow was imposed on the inlet.

Studies of the strain rate, first normal stress difference (N_1) and the extensional viscosities were analysed and compared to the strain rate of a 45 degree contraction. The extension through the hyperbolic contraction was constant compared to the 45 degree contraction making it suitable in the Hyperbolic Contraction Flow measurement system.

ACKNOWLEDGMENTS

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