Studies of the Rheological Properties of Drilling Fluids

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ABSTRACT

The methodology and results of investigations of rheological properties of drilling fluids on the basis of rotational viscosimetry data have been described. Special attention has been paid to rheological properties of biviscosity fluids and applied aspects of determination of state equations.

INTRODUCTION

Technological processes in oil and gas industry are closely connected with the use of non-Newtonian fluids. Efficient management of these processes requires precise determination of rheological parameters and influence of any factors upon them. For this purpose the rotational viscosimeters are most widely used but the simplified methodologies for viscosimetry data treatment do not allow using all possibilities of the measuring equipment.

The Couette flow in a gap between two coaxial cylinders of rotational viscosimeters is described by equation

$$\omega = \frac{1}{2}\int_{\tau_{out}}^{\tau} \frac{\dot{\gamma}(\xi)}{\xi} d\xi \tag{1}$$

where ω is angular speed of rotation of outer cylinder, τ and τ_{out} are shear stresses on inner and outer cylinders, $\dot{\gamma}(\tau)$ is fluid rheological model, $\dot{\gamma}$ is shear rate gradient.

The dependence between stresses on the outer and inner cylinders is determined by the following relation:

$$\tau_{out} = \begin{cases} \alpha^2\tau, \text{ if } \tau \geq \tau_0/\alpha^2; \\ \tau_0, \text{ if } \tau_0 \leq \tau < \tau_0/\alpha^2, \end{cases} \tag{2}$$

where τ_0 is yield stress, $\alpha = R_1/R_2$, and R_1 and R_2 are the radii of the inner and outer cylinders.

In Eq. 1 the rheologically stationary models are used which permits clear analytical solution of $\dot{\gamma} = \dot{\gamma}(\tau)$.

METHODOLOGY

The proposed methodology[1] for rotational viscosimetry data treatment is based on exact solution of the Eq. 1 and considers informational relevance of the experiments.

The aim of rotational viscosimetry data processing is formalized as the task of search of $\hat{\nu}$ index of rheological model and its parameters $\hat{a}_\nu$ in some a priori known class of ϑ models

$$\tau = \begin{cases} \text{either } A(\omega, a_1) + \varepsilon_1; \\ \text{.........................} ; \\ \text{or } \quad A(\omega, a_\nu) + \varepsilon_\nu, \ \nu \in \vartheta; \\ \text{.........................} , \end{cases} \tag{3}$$

where τ is vector of measured shear stresses at angular rates ω, $A(\omega, a_v)$ is operation of direct task of rotational viscosimetry Eqs. 1 and 2, a_v is vector of rheological properties, ν is index of rheological model; ε_v is vector of centered normal random component, caused by measurement inaccuracies.

The solution of the task Eq. 3 is built using the principle of maximum of likelihood function and is realized with the help of the following procedures

$$\min_{\nu \in \vartheta} \| C^{-1/2} \left(A(\omega, a_\nu) - \tau \right) \| \Rightarrow (\hat{a}_\nu, \hat{\nu}); \quad (4)$$

$$\min_{\nu \in \vartheta} \left\{ \begin{array}{l} \hat{\sigma}_\nu^2 = \dfrac{1}{N - r_\nu} \times \\ \times \sum_{i=1}^{N} \left(A(\omega_i, \hat{a}_\nu) - \tau_i \right)^2 \end{array} \right\} \Rightarrow \hat{\nu}, \quad (5)$$

where N is number of rates of cylinder rotation, r_ν is number of rheological parameters being evaluated, C is matrix of covariations of random component in Eq. 3.

According to Eq. 4 at first for each model $\nu \in \vartheta$ evaluations $\hat{a}_\nu$ of rheological properties are built, and then according to Eq. 5 rheological model $\hat{\nu}$ is evaluated.

Contrary to other existing methods, methodology allows more exact selection of the proper rheological model and precise estimation of its parameters[2].

Methodology allows to select the most appropriate model in the class of the rheologicaly stationary models: Newton, Bingham, Ostwald, Herschel-Bulkley, Schulman-Casson and biviscosity models, estimate their rheological properties and build the estimation error covariance matrix O.

$$O = \left(A'(\omega, \hat{a}_{\hat{\nu}}) C^{-1} A'^{T}(\omega, \hat{a}_{\hat{\nu}}) \right)^{-1}, \quad (6)$$

where $A'(\omega, \hat{a}_{\hat{\nu}})$ is matrix of derivatives of rheological parameters and transposed to it is matrix $A'^{T}(\omega, \hat{a}_{\hat{\nu}})$.

The biviscosity model[3] is considered as any combination of rheological models, which describes rheological properties of fluid in different ranges of shear rates.

$$\dot{\gamma} = \begin{cases} \dot{\gamma}\left(\tau, a^{(1)}\right), \tau \le \tau^*; \\ \dot{\gamma}\left(\tau, a^{(2)}\right), \tau > \tau^*, \end{cases} \quad (7)$$

where $a^{(1)}, a^{(2)}$ are rheological properties accordingly for the low and high gradients of shear rates, τ^* is boundary shear stress, calculated from the equation $\dot{\gamma}\left(\tau^*, a^{(1)}\right) = \dot{\gamma}\left(\tau^*, a^{(2)}\right)$.

For the selection of the most appropriate rheological model, which describes fluid in the entire technological process (under the influence of parameters of state or any additives) the procedure of batch processing is proposed[4]. In this case Eqs. 4 and 5 are generalized as follows:

$$\min_{\nu \in \vartheta} \| A(\omega, a_{j\nu}) - \tau_j \| \Rightarrow \hat{a}_{j\nu},\ j = \overline{1, z}; \quad (8)$$

$$\min_{\nu \in \vartheta} \left\{ \begin{array}{l} \hat{\sigma}_{c\nu}^2 = \dfrac{1}{z(N - r_\nu)} \times \\ \times \sum_{j=1}^{z} \sum_{i=1}^{N} \left(A(\omega_i, \hat{a}_{j\nu}) - \tau_{ji} \right)^2 \end{array} \right\} \Rightarrow \hat{\nu}, \quad (9)$$

where $\hat{\sigma}_{c\nu}^2$ is dispersion of adequacy for the whole number of experiments z with different values of influencing factors.

The polynomial and spline models are used for the describing of any factors influence on the rheological properties.

The spline functions are represented according to V.A. Vasylenko[5]

$$p(\mathbf{x}) = \sum_{j=1}^{z} b_j G_{m,z}\left(\mathbf{x}-\mathbf{x}_j\right) + \sum_{j=1}^{q_3} b_{z+j} (\mathbf{x})^{\boldsymbol{\alpha}_j},$$

where b_j, b_{z+j} are parameters of analytical spline representation, z is number of experimental points,

$$G_{m,z}(\mathbf{x}-\mathbf{x}_j) = \begin{cases} \left\|\mathbf{x}-\mathbf{x}_j\right\|^{2m-z} \ell n \left\|\mathbf{x}-\mathbf{x}_j\right\|, & \text{if } z \text{ is even;} \\ \left\|\mathbf{x}-\mathbf{x}_j\right\|^{2m-z}, & \text{if } z \text{ is odd,} \end{cases}$$

$$\left\|\mathbf{x}-\mathbf{x}_j\right\| = \left(\sum_{i=1}^{z} \left(x_i - x_{ij}\right)^2 \right)^{1/2},$$

$q_3 = (z+m-1)!/(m-1)!z!$,

$\mathbf{x}_j = \left(x_{1j}, x_{2j}, \ldots, x_{zj}\right)^T$ is vector of influence factors in the experiment j,

$\boldsymbol{\alpha}_j = \left(\alpha_{1j}, \alpha_{2j}, \ldots, \alpha_{zj}\right)$ is multiindex, $(\mathbf{x})^{\boldsymbol{\alpha}_j} = (x_1)^{\alpha_{1j}} (x_2)^{\alpha_{1j}} \ldots (x_z)^{\alpha_{zj}}$ and m is parameter of variational functional.

THE STUDY

As it was mentioned in methodology description, biviscosity models have been included into the class of rheological models, among which the most adequate one is being selected. First-priority task was to find out whether fluids being described by these models do exist. The search of such

Table 1. Results of rotational viscometry data treatment where most appropriate is biviscosity rheological model

Studied fluid	Device gap	Most appropriate biviscosity rhelogical model	Model parameters					Boundary shear stress τ^*, Pa	Dispersion $\hat{\sigma}_v^2$, Pa^2	Dispersion for other models $\hat{\sigma}_v^2$, Pa^2		
			shear rates	τ_0, Pa	k, Pa·s^n	η, Pa·s	n			Herschel-Bulkley	Ostwald	Bingham
1	0.79384	Bingham & Ostwald	low	1.104	–	0.1994	–	12.08	0.0108	0.0655	0.0573	0.9857
			high	–	0.6555	–	0.7167					
2	0.9365	Ostwald & Newton	low	–	1.4680	–	0.4189	21.82	0.7674	2.5660	5.9580	1.7950
			high	–	–	0.0379	–					
3	0.9365	Ostwald & Bingham	low	–	0.3202	–	0.4572	4.94	0.0257	0.0771	0.1182	0.2141
			high	1.775	–	0.0083	–					
4	0.9365	Ostwald & Ostwald	low	–	1.5920	–	0.2600	6.01	0.0907	0.0920	0.6272	0.5939
			high	–	0.3690	–	0.5610					

Note. τ_0 is yield point, k is consistency index, η is plastic viscosity and n is flow-behavior index.

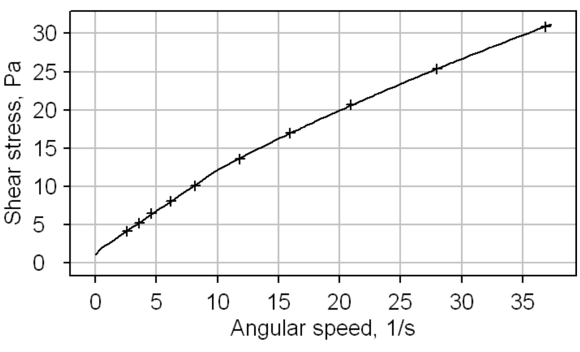

Figure 1. Rheogram for studied fluid 1

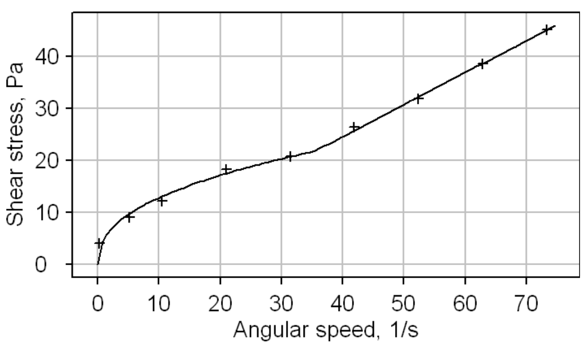

Figure 2. Rheogram for studied fluid 2

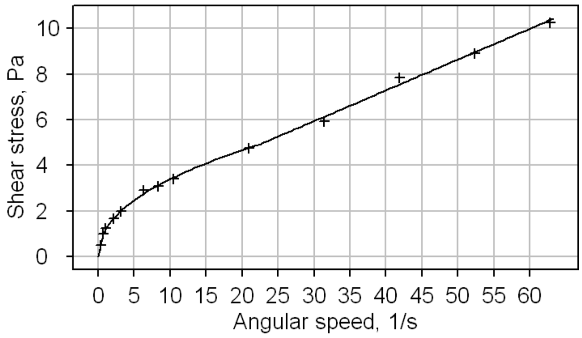

Figure 3. Rheogram for studied fluid 3

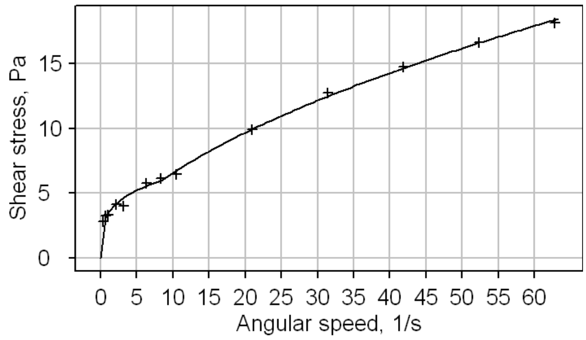

Figure 4. Rheogram for studied fluid 4

fluids was carried out by treatment of rheometry data, obtained by our colleagues and us and taken from literature. In table 1 and on figures 1–4 the most vivid results for different biviscosity models, which are as follows: studied fluid 1 is CMC–water solution (Sample 33[6]), studied fluid 2 is cement slurry with alumina-silicate microspheres, studied fluid 3 and 4 are bentonite–lignite suspensions (Sample 5 and Sample 7 respectively[6]).

It should be mentioned that biviscosity fluids among drilling fluids occur rather frequently, but the fraction of such fluids is difficult to evaluate while it depends on the type of fluid. Another important issue for the evaluation of parameters of biviscosity fluids is the number of rates of viscosimeter, used to make measurements.

The rheological properties of drilling fluids are more sensitive to influence of temperature, pressure and additives then other parameters. So, while selecting drilling fluids formulation, the attention must be previously paid on rheological properties especially when the aim is to select the most thermostable prescription.

In the case of selection drilling mud formulation the thermostability means: mud properties before and after heating must be in specified boundaries and the parameters changes must be minimal.

In table 2 there are rotational viscosimetry data, obtained according to the plan of experiment, the aim of which was the selection of optimal formulation of humate-biopolimer drilling mud.

Table 2. Rotational viscometry data for humate-biopolimer drilling mud

Experiment	Values of factors			Angles of turn (degree) at rotating rates (rpm)													
				before heating							after heating						
	CAR, %	Polypac UL, %	Duovis, %	6	30	60	100	200	300	600	6	30	60	100	200	300	600
1	2	0.1	1	51	50	55	60	69	75	85	5	14	20	26	35	41	52
2	2	0.6	0.1	2	4	5	7	12	15	25	0	1	2	3	5	6	11
3	12	0.1	0.1	4	8	11	15	22	28	46	1	4	5	7	10	12	19
4	12	0.6	1	41	59	72	81	102	124	156	34	43	51	58	66	72	92
5	2	0.1	0.1	2	3	4	5	6	8	12	0	0	0	1	2	2	3
6	2	0.6	1	42	56	64	73	87	98	117	7	17	24	30	40	46	60
7	12	0.1	1	50	65	75	85	105	117	148	40	52	58	65	74	81	92
8	12	0.6	0.1	3	6	8	12	20	25	42	2	3	5	7	11	14	22
9	13.08	0.35	0.55	26	37	46	56	74	89	116	15	23	27	31	38	44	57
10	0.92	0.35	0.55	15	21	25	29	36	42	54	7	13	18	21	27	31	37
11	7	0.65	0.55	19	28	38	47	65	78	108	11	18	23	27	34	39	50
12	7	0.05	0.55	18	26	30	35	44	51	72	7	14	18	21	26	30	38
13	7	0.35	1.10	60	77	87	91	111	121	147	42	51	58	64	74	81	96
14	7	0.35	0	0	1	2	3	5	7	13	0	0	1	2	3	5	8
15	7	0.35	0.55	18	26	32	38	49	58	78	11	18	23	25	32	36	46

Note. CAR is coal-alkali reagent. Device constant is 0.4788 Pa/degree, device gap is 0.9365.

Thermostating of drilling fluid in each experiment has been carried out for 4 hours in autoclave (with the capacity of 400 cm^3) in roller oven at the temperature of 120 °C. As a result of treatment of presented data, parameters of rheological models in each point of experiment plan have been evaluated and the most adequate among them have been selected (Table 3).

Table 3. Most appropriate rheological models at experimental points

Exp.	Most appropriate models and dispersion $\hat{\sigma}_v^2$, Pa^2			
	before heating		after heating	
1	Biviscosity Bingham & Ostwald	0.7413	Ostwald	0.6522
2	Herschel-Bulkley	0.0349	Ostwald	0.0206
3	Schulman-Casson	0.0476	Ostwald	0.0463
4	Herschel-Bulkley	1.2850	Herschel-Bulkley	0.6888
5	Herschel-Bulkley	0.0238	Ostwald	0.0403
6	Herschel-Bulkley	0.1327	Biviscosity Bingham & Ostwald	0.3617
7	Herschel-Bulkley	0.1226	Herschel-Bulkley	0.0921
8	Herschel-Bulkley	0.0822	Herschel-Bulkley	0.0239
9	Herschel-Bulkley	0.4406	Herschel-Bulkley	0.1097
10	Herschel-Bulkley	0.0100	Biviscosity Ostwald & Ostwald	0.1704
11	Herschel-Bulkley	0.2164	Herschel-Bulkley	0.009
12	Herschel-Bulkley	0.3087	Ostwald	0.0675
13	Herschel-Bulkley	1.1040	Herschel-Bulkley	0.0254
14	Ostwald	0.0078	Ostwald	0.0366
15	Herschel-Bulkley	0.0042	Ostwald	0.1136

But it is clear that it's practically impossible to use several rheological models for the analysis of influence of reagents upon rheological properties of drilling fluid. Thus, according to Eqs. 8 and 9 dispersion of adequacy for the whole number of experiments was estimated (Table 4). Table 4 contains estimations of dispersion of adequacy just for five rheological models, because for other models it wasn't possible to make such estimation while data could not be described with these models in some experimental points. From Table 4 one can make a conclusion that in general Herschel-Bulkley model is the most adequate for the description of results of experiments, the evaluations of parameters of which are given in Table 5.

Table 4. Generalized functional for model selection

Rheological model	Dispersion of adequacy $\hat{\sigma}_{cv}^2$, Pa^2	
	before heating	after heating
Herschel-Bulkley	0.605	0.3613
Schulman-Casson	1.696	-*
Ostwald	2.04	0.3885
Bingham	9.227	5.143
Newton	218	89.5

Note. * - data could not be described with this model in some experimental points.

Table 5. Results for Herschel-Bulkley model

Exp.	Model parameters						Δa
	before heating			after heating			
	τ_0, Pa	k, $Pa \cdot s^n$	n	τ_0, Pa	k, $Pa \cdot s^n$	n	
1	20.64	0.3502	0.575	0	1.2920	0.4260	100
2	0.60	0.0533	0.7714	0	0.0266	0.7597	47
3	1.33	0.1255	0.7328	0.053	0.1601	0.5762	80
4	11.65	2.4150	0.4649	11.06	1.7230	0.4152	50
5	0.80	0.0361	0.7045	0	$5.6 \cdot 10^{-17}$	5.4980	34
6	10.56	3.6700	0.3559	0	1.613	0.4125	128
7	16.62	2.0920	0.462	7.34	6.005	0.2538	87
8	0.74	0.0989	0.7589	0.35	0.0834	0.6908	50
9	6.68	1.4740	0.5007	4.05	1.117	0.4300	79
10	4.52	0.7883	0.4692	0	1.675	0.3386	34
11	4.53	1.0560	0.5439	0.97	1.745	0.3654	82
12	7.10	0.3866	0.607	0	1.661	0.3398	68
13	20.51	2.7670	0.4074	13.69	2.205	0.3779	44
14	0	$2.8 \cdot 10^{-9}$	3.136	0	$2.1 \cdot 10^{-16}$	5.4470	25
15	5.48	0.8145	0.5231	1.16	1.792	0.3468	92

Rheological properties refer to vector quantities, specified change of which can be estimated on the basis of multidimensional function of probability density[7]

$$\Delta a = \left[\left(a_0 - a_1 \right)^T O^{-1} \left(a_0 - a_1 \right) \right]^{1/2}, \qquad (10)$$

where a_0, a_1 are vectors of rheological properties of drilling fluid correspondingly before and after thermal influence.

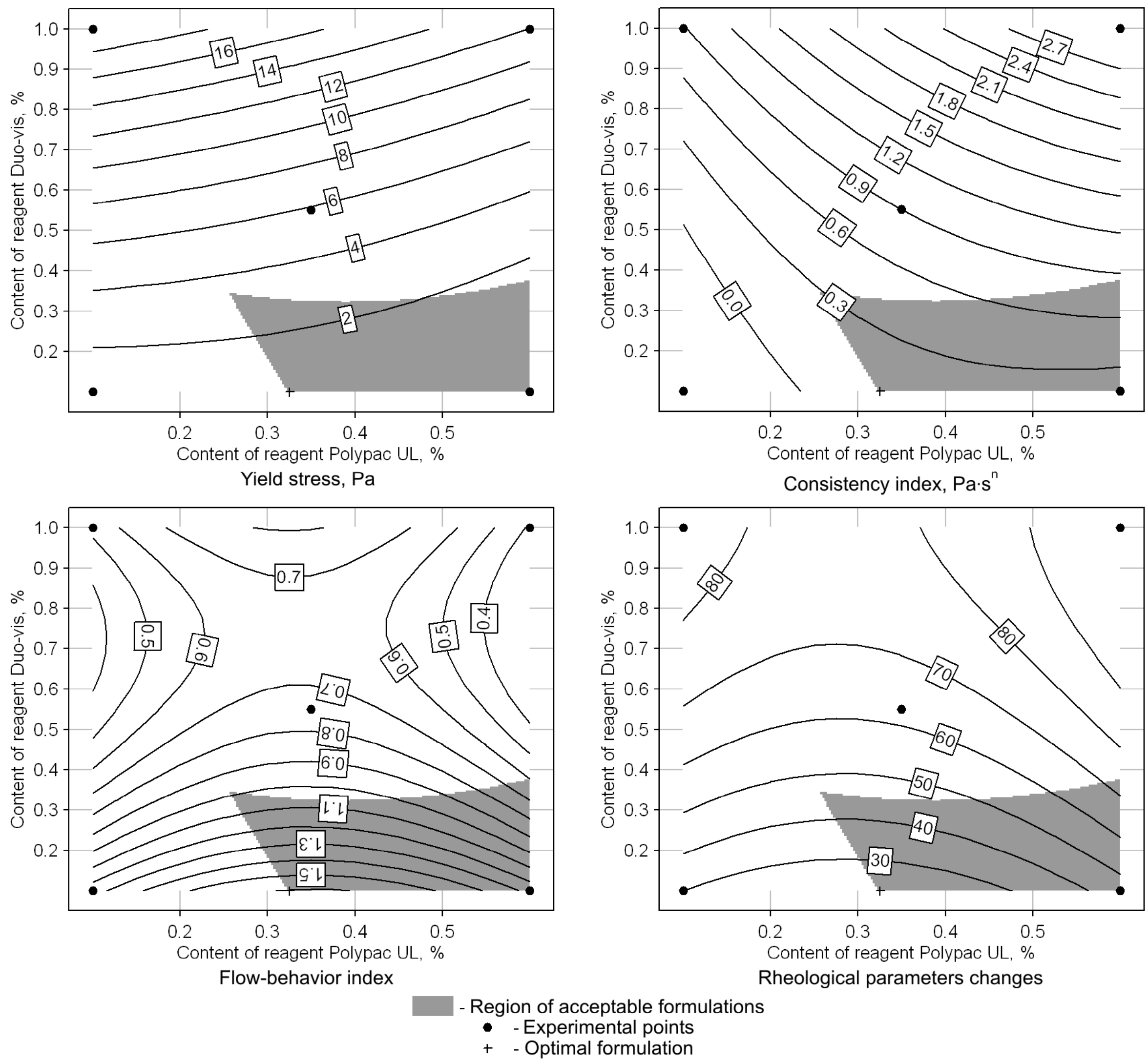

Figure 5. Influence of reagents on the rheological properties of humate-biopolimer drilling mud

According to Eq. 10 estimations of change of rheological properties in the course of thermostating have been built, which are given in Table 5.

While experiment was carried out according to orthogonal central composition plan, for the description of its parameters second-degree polynoms have been used. In figure 5 dependencies of rheological parameters upon concentrations of reagents in drilling fluid at the specified CAR value of 3.5 % are represented by isolines. Analogical dependencies were built for other properties of drilling fluid, upon values of which were imposed interval restrictions and which allowed estimating region of acceptable formulations.

Region of acceptable formulations is quite broad, that’s why selection of optimal formulation of drilling fluid requires formalization of optimization criterion. In this case, the aim is to select the most stable formulation in terms of rheological parameters changing, which is ensured by minimization of criterion of Eq. 10. Figure 5 represents dependence of criterion of Eq. 10 upon concentrations of reagents and optimal drilling fluid formulation, obtained due to its use.

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