Study of Flow and Blockage of Highly Concentrated Granular Suspensions Under Squeeze Test Conditions

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ABSTRACT

Modelling the flow behaviour of granular suspensions represents a great challenge since they are characterized by a rather complex rheological behaviour. Moreover macroscopic heterogeneities may be induced by the flow due to eventual relative motion between the liquid and the granular phases under certain conditions. Hence, in order to model the flow of such material, one has to explicitly consider that it comprises at least two phases. In the present approach we consider that these phases are continuous and behave as power-law fluids. The coupling between the two media is accounted for using a generalized Darcy's filtration law. The model is solved using the FE method in the case of squeeze flow geometry.

INTRODUCTION

A number of industrial materials are composites made-up of a continuous phase (matrix) in which particles of different forms are dispersed at different concentrations in order to improve the properties of the matrix. This includes polymer and metallic short fibre composites, cementitious materials, etc. In order to take full advantage of the effect of the inclusions regarding the effective composite properties, including mechanical, thermal, electrical properties, etc., the particles have to be homogeneously distributed throughout the matrix. These materials are processed while they are in liquid or paste state. Hence, the problem of composite processing modelling boils down to that of flow of (generally concentrated) suspensions of solid particles in complex fluids.

Several studies have been reported in the literature pointing out that concentrated suspensions or pastes become heterogeneous in complex flows. For example, it is well known that non-uniform shear flows induce particle migrations in concentrated suspensions[1]. The origin of this phenomenon is well understood and attributed to irreversible interactions (collisions) among the particles[2]. This process is diffusive[2] and can be shown to be negligible in the flow situation considered in our study. Here we focus on the problem of flow-induced heterogeneities under squeeze flow conditions. Indeed, several studies have shown that concentrated suspensions can become heterogeneous in squeeze flows[3-5]. Squeeze-flow-induced heterogeneities have been attributed to the filtration of the fluid phase through the porous media made-up by the solid particles. This has been interpreted[5] in terms of the competition between the flow of the suspension as a whole and the Darcy filtration of the fluid phase. The two phenomena take place at different time scales depending upon different properties of the suspension, including its rheological properties, those of the fluid phase and the permeability of the granular skeleton. A qualitative discussion, in which these two characteristic times have been compared, was reported[4-5].

In the present study a more quantitative investigation regarding flow-induced heterogeneities in concentrated granular suspensions is developed using a multiphasic model. The model is solved using the FE method in the case of the squeeze test. Confrontation our model with experiments will be reported elsewhere.

MODELLING

Assuming the hypothesis that one can define a Representative Elementary Volume (REV), a suspension can be described as a superposition of at least two different continuous media. Here we consider only two phases (a solid phase indexed "s" and a fluid phase indexed "f"), but the approach can be easily generalized to more phases. At each point in space, the phases coexist with their corresponding volume fractions ϕ_s and ϕ_f. Each phase is characterized by its macroscopic velocity field (c = s or f) its stress tensor field , its strain tensor field and its own boundary conditions.

<u>Mass balance</u>

The phases are assumed to be incompressible and there are no source terms:

$$\frac{\partial \phi_f}{\partial t} + \operatorname{div}\left(\phi_f \overrightarrow{v_f}\right) = 0 \quad \text{and} \quad \frac{\partial \phi_s}{\partial t} + \operatorname{div}\left(\phi_s \overrightarrow{v_s}\right) = 0 \tag{1}$$

<u>Momentum balance</u>

For the sake of simplicity, volume forces such as gravity are ignored; however they can be included in a straightforward manner. Due the high viscosity of the materials considered here, inertia is generally negligible compared to the viscous forces (small Reynolds numbers). The conservation equation of momentum can be then written for each phase as:

$$\overrightarrow{\operatorname{div}}\left(\underline{\underline{\sigma_f}}\right) + \overrightarrow{\pi_f} = \vec{0} \quad \text{and} \quad \overrightarrow{\operatorname{div}}\left(\underline{\underline{\sigma_s}}\right) + \overrightarrow{\pi_s} = \vec{0} \tag{2}$$

In these equations $\vec{\pi}_{f,s}$ is the momentum exchanged by the phases. Here, we assume isothermal conditions, so it is not necessary to write down the equations of energy conservation.

<u>Constitutive equation for the fluid phase</u>

The fluid phase in 'real' materials is in general complex (polymer melts, colloidal suspensions, etc.). Such fluids have non-linear behaviours. Without loss of generality, we can assume power-law behaviour for the fluid phase. A yield stress can be further introduced without particular difficulty. The constitutive equation for the fluid phase writes then:

$$\underline{\underline{\sigma}}_f = -\phi_f p \underline{\underline{I}} + 2\eta_f \underline{\underline{D}}_f, \quad \text{with} \quad \eta_f = K_f \left(\sqrt{2\operatorname{tr}\left(\underline{\underline{D}}_f^{\,2}\right)}\right)^{m_f - 1} \tag{3}$$

In Eq. (3) p is the pressure and K and m, are the consistency and the fluidity index of the fluid phase, respectively.

<u>Constitutive equation for the solid phase</u>

We also assume that the solid phase behaves as a power-law fluid. Actually, a number of experimental results reported in the literature showed that concentrated suspensions can be de-scribed by the Herschel-Bulkley rheological model; that is a power-law fluid with a yield stress. For the sake of simplicity, we ignore for the moment the yield stress term. Assuming that, we consider the flow behaviour beyond the yield stress. The constitutive equation for the solid phase is then :

$$\underline{\underline{\sigma}}_s = -\phi_s p \underline{\underline{I}} + 2\eta_s \underline{\underline{D}}_s \quad \text{with} \quad \eta_s = K_s \left(\sqrt{2\mathrm{tr}\left(\underline{\underline{D}}_s^{\,2} \right)} \right)^{m_s - 1} \tag{4}$$

where K_s and m_s refer to, respectively, the consistency and the fluidity index of the solid phase.

A number of theoretical or experimental approaches showed that the consistency of a granular suspension diverges at the volume fraction of maximum packing ϕ_m [6-7]. The Krieger-Dougherty model gives a reasonable description of the behaviour of concentrated suspension near ϕ_m. This model is used here to account for the concentration dependence of the consistency. That is:

$$K_s = K_0 / \left(1 - \frac{\phi_s}{\phi_m} \right)^{\alpha} \quad \text{with} \quad \alpha > 0, \tag{5}$$

The parameter is chosen so to recover the Einstein's relationship in the dilute regime in the case of spherical particles; that is $K_s = K_0 \left(1 + 2.5\phi_s \right)$. Then we take $\alpha = 2.5\phi_m$. We assume here that the particles are mono disperse and spherical. In this case, for a random packing we have: ϕ_m=0.64.

Coupling between the phases

In previous (monophasic) modelling of suspension flows, it was generally assumed that at each space-point the fluid and the solid move at the same local velocity. In a multiphasic approach it is assumed that there is a relative motion between the different phases. This will lead to a pressure drop that can be taken account using the Darcy's law. Since the fluid phase is not Newtonian, the classical Darcy law has to be generalized to such fluids. We use here the model of Fadili et al.[8] for power-law fluids:

$$\vec{\pi}_s = -\vec{\pi}_f = k \left(\sqrt{\left(\vec{v}_f - \vec{v}_s \right)^2} \right)^{m_f - 1} \left(\vec{v}_f - \vec{v}_s \right) + p\,\overrightarrow{\mathrm{grad}}\left(\phi_s \right) \tag{6}$$

The interaction coefficient k is the ratio of the viscosity of the fluid phase to the Darcy permeability B of the granular skeleton relative to a Newtonian fluid. The Darcy permeability depends only upon the geometrical properties of the granular skeleton. The Kozeny-Carmen model is used here to relate the permeability to the geometrical properties of the suspension:

$$B = b \frac{\phi_f^{\,3}}{\left(1 - \phi_f \right)^2} \quad \text{with} \quad b = \frac{1}{2CT^2S^2} \cong \frac{D^2}{100} \tag{7}$$

where T is the tortuosity (related to the connectivity of the pores) of the porous media made-up by the particles, S is its specific area and D is the average diameter of the pores (or particles). The coefficient is related to the form of the pores.

Boundary conditions

To illustrate, we consider here the case of a squeeze test. The material is squeezed out between two parallel plates at a controlled velocity V_0 (Fig. 1) and the normal force experienced by one of the plates is recorded.

Since we deal with low-Reynolds number conditions, for each phase, we can assume that there is no relative motion at the wall boundaries. At the free boundaries, the two phases are assumed to be in equilibrium with the external environment. That is (Fig. 1):

$$\vec{V}_f = \vec{V}_s = \vec{V}_0 \ , \ \underline{\underline{\sigma}}_f \vec{n} = -\phi_f p_a \vec{n} \qquad \underline{\underline{\sigma}}_s \vec{n} = -\phi_s p_a \vec{n} \tag{8}$$

where $\vec{n}$ is a normal unit vector oriented in the outward direction and p_a the atmospheric pressure.

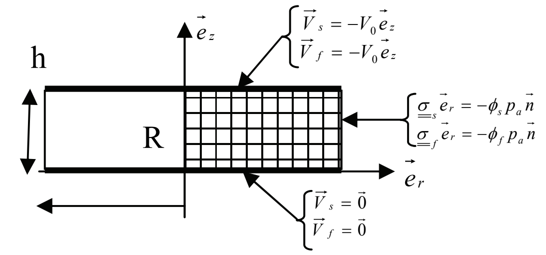

Figure 1. Boundary conditions used to simulate to squeeze test.

NUMERICAL SOLVING: CASE OF THE SQUEEZE TEST

We consider a squeeze flow of a granular highly concentrated suspension between two parallel and circular plates moving towards each other at the same controlled velocity. The flow geometry is axisymmetric, so the 3D problem reduces to a 2D problem (see Fig. 1). Since the flow is unsteady, we adopt an incremental approach through three steps[9]:

(i) At a first step we assume that the volume fraction is known and we solve the flow problem, which consists of a simple Stokes problem. We use a finite element method for a mixed formulation velocity-pressure. The Newton-Raphson method is used to solve for the non-linearities. In order to avoid numerical blockage, we use a quadratic interpolation for the velocity and a linear one for the pressure.

(ii) In a second step, the volume fraction is updated by an implicit algorithm and a Taylor-Galerkin approach to solve for the mass conservation. The numerical difficulty here stems from the hyperbolic form of this equation.

(iii) In the last step, after the time increment, the meshing is convected.

RESULTS AND DISCUSSIONS

Fig. 2 represents the evolution of the squeeze force as a function of the instantaneous distance between the plates for different velocities. The suspension properties chosen here correspond to those of a typical cement paste. This figure shows that there is a radical change of the behaviour of the squeeze force when the velocity is decreased. At relatively high velocity (above 1mm/s in Fig. 2), the force smoothly increases when the gap thickness decreases. This is what is expected for a viscous fluid. Below a certain "critical" velocity (1 mm/s in the case of Fig. 2) a divergence of the squeeze force is obtained for a certain gap thickness. This indicates that we have blockage of the flow. Fig. 3 shows that the blockage takes place at higher gap thicknesses for more concentrated suspensions.

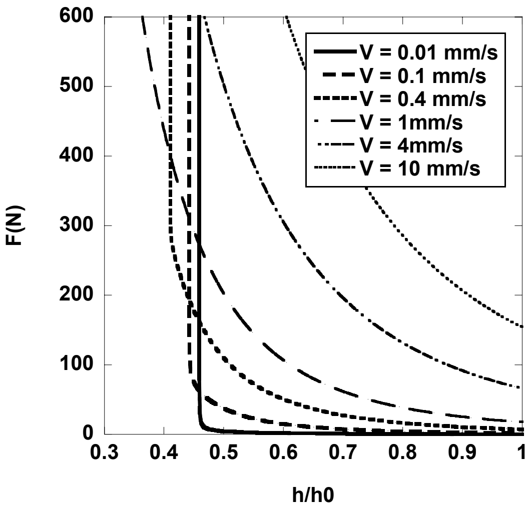

Figure 2. Squeeze force against rescaled instantaneous gap thickness for different velocities :

$K_s = 100\ Pa.s^{m_s}; m_s = 0.6; K_f = 10\ Pa.s^{m_f}; m_f = 1.0; k = 10^7 Pa.s^{m_f}.m^{-2}; \phi_s = 0.3$

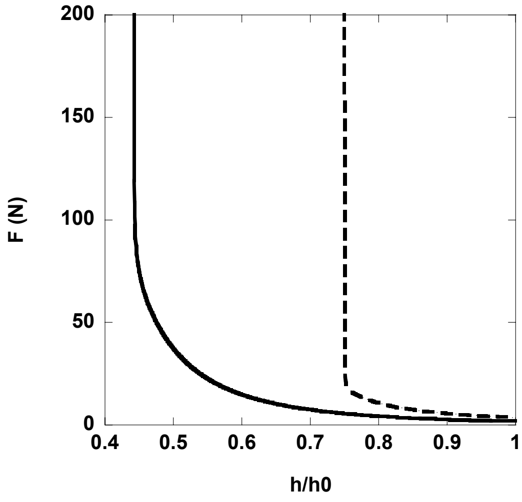

Figure 3. Squeeze force against rescaled instantaneous gap thickness for velocity 0.1 mm/s (solid line $\phi_s = 0,3$ and dashed line for $\phi_s = 0,5$)

$K_s = 100 Pas^{m_s}; m_s = 0.6; K_f = 10 Pas^{m_f}; m_f = 1.0; k = 10^7 Pas^{m_f}.m^{-2}$

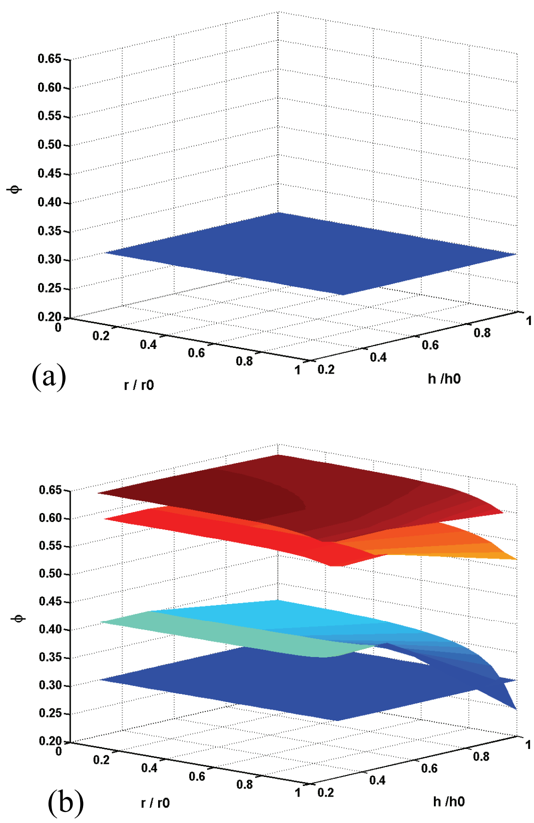

Figure 4. Evolution of the volume fraction field for different instantaneous gap thicknesses: (a) at a relatively high velocity (V=10 mm/s); (b) at small velocity (V=0.01 mm/s).

Fig. 4 represents the evolution of the solid volume fraction pattern for the extreme cases of high and small velocity. These results correlate well with the behaviour of the squeeze force. At high velocities the material remains homogeneous, while at small velocities we observe an increase of the solid concentration around the central part of the set-up at different stepped times (h/h0).

CONCLUSIONS

We have presented a multiphasic modelling to simulate the processing of highly concentrated granular suspensions. It was shown that such model is able to capture flow-induced heterogeneities and blockage that may take place during flow of such materials.

The influence of the geometry of the flow and that of the rheological and geometrical properties of the phases deserves to be considered in details. This issue will be discussed and comparison with experiments will be performed in a forthcoming publication.

REFERENCES

1. Leighton, D. and Acrivos, A. (1987), "Shear-induced migration of particles in concentrated suspensions", *J. Fluid Mech.*, 181, 415.

2. Cunha, F.R. da and Hinch, E.J. (1996), "Shear-induced dispersion in a dilute suspension of rough spheres", *J. Fluid Mech.*, **309**, 211-223.

3. Sherwood, J.D. (2002), "Liquid-solid relative motion during squeeze flow of pastes", *JNMFM* , **104**, 1-32.

4. Chaari, F., Racineux, G., Poitou, and A., Chaouche, M. (2003), "Rheological behavior of sewage sludge and strain-induced dewatering", *Rheol. Acta*, **42**, 273-279.

5. Delhaye, N., Poitou, A., Chaouche, M. (2000), "Squeeze flow of highly concentrated suspensions of spheres". *JNMFM*, **94** (1), 67-74.

6. Krieger, I.M., and Dougerthy, T.J. (1959), "A mechanism for non-Newotonian flow in suspensions of rigid spheres", *Tran. Soc. Rheol.*, **3**, 137-152.

7. Quemada, D. (1977), "Rheology of concentrated disperse systems and minimum energy dissipation principle viscosity, concentration, relationship", *Rheol. Acta*, **16**, 82-94.

8. Fadili, A., Tardy, P.M.J., and Pearson, J.R.A. (2002), "A 3D filtration law for power-law fluids in heterogeneous porous media", *JNNFM*, **106**, 121-146.

9. Poitou, A, and Racineux, G. (2001), “A squeezing experiment showing binder migration in concentrated suspensions”, *J. Rheol.,* **45**(3), 609-625.