Some Provocative Differences Between Planar and Axisymmetric Flows in the Case of Some Elastic Liquids

Ken Walters

Professor Ken Walters, FRS, Department of Mathematics,
University of Wales, Aberystwyth
Ceredigion, SY23 3BZ, UK.

In this lecture I want to discuss some experimental results that have intrigued me. Some of the experiments were carried out a decade or two ago, but others are more recent. They concern complex flows of non-Newtonian elastic liquids.

I shall provide experimental evidence that, in some cases, whether or not the liquid is shear-thinning is of crucial importance. I must emphasise at the outset that this unusual state of affairs exists only in isolated circumstances.

To add to the intrigue, I also want to investigate the similarity, or otherwise, between planar and axisymmetric versions of the same flow. Sometimes they show the expected similarities and sometimes not, and, interestingly, the existence or absence of shear thinning is of crucial importance when provocative differences are in evidence.

I must admit that a number of my preconceived ideas have been shattered by the experimental results I will show you. It has been a learning curve for me and I shall leave you with a number of intriguing questions, many of which are far from being resolved.

But first I must be quite specific about my basic thesis. I need to make it crystal clear what I mean when I talk about planar and axisymmetric versions of the same flow.

As an example, consider contraction flows. The contractions are abrupt in both cases and, in the axisymmetric case we have pressure-driven flow from one capillary into another of smaller diameter. In the planar version, the third dimension is considered long enough for the flow to be deemed two-dimensional. We shall be basically concerned with the pressure differences that are required to produce given flow rates, together with the flow structure. In particular, we are interested in the existence or otherwise of lip vortices and also whether so-called vortex enhancement is present.

I show you a typical picture of the flow structure. This happens to be for a 0.25% aqueous solution of polyacrylamide in a 4:1 *planar* contraction. Both a lip vortex and a salient corner vortex are in evidence.

The next picture illustrates what we mean by vortex enhancement. This time, we have an 8:1 *axisymmetric* contraction and the liquid is a constant-viscosity Boger fluid.

Whether or not the lip vortex (when it is present) or the salient corner vortex grows in the development of vortex enhancement is a crucial question.

So much for flow structure. Now a word about the dynamics. Vortex enhancement is invariably accompanied by a relatively high pressure gradient to produce a given flow rate, and this is usually studied through the so called ‘Couette correction’.

Let me now introduce you to the fascinating phenomena associated with the ‘splashing’ experiment.

In the axisymmetric case, a *sphere* is dropped from some distance above the free surface of a liquid and a rich sequence of events unfolds. The initial crater, the crown structure and the vertical jet (with the possibility of distinct satellite drops) are all features that can occur in a single experiment. The vertical jet can often reach extravagant heights. We call it the Worthington jet, after the British scientist who first investigated the effect over one hundred years ago.

In the *planar* version of the splashing experiment, a long horizontal *rod* is released onto the free surface of the liquid.

Let make it quite clear what I mean by the presence or absence of 'shear thinning'. I probably don't need to labour this, but it does provide me with an opportunity to discuss steady simple shear flow.

Here, σ_{ik} is the stress tensor and γ a constant shear rate. The stress components can be written in terms of three so-called viscometric functions; a shear stress, (or equivalently a viscosity) and two normal stress differences N_1 and N_2. For the vast majority of elastic liquids, we have so-called 'shear thinning', with the viscosity falling as the shear rate is increased. I show you data for a typical polymer solution, one that we shall be studying in a moment.

Now, in this presentation, I shall be considering the behaviour of some aqueous polymer solutions and also so-called Boger fluids.

For this small but important sub-class of Boger fluids, the viscosity is essentially constant, although the fluids can still be 'highly elastic'. These are invariably constructed by dissolving a small concentration of a high molecular weight polymer like polyacrylamide in a fairly viscous Newtonian solvent like maltose syrup.

For completeness and future reference, I must remind you of another important rheometrical flow called 'uniaxial extension'. Here, the so-called extensional viscosity is a function of the strain rate $\in$. We usually find it convenient to talk in terms of the Trouton Ratio, which happens to be 3 for a Newtonian liquid. Elastic liquids can often have Trouton ratios far in excess of the Newtonian value.

There is of course a planar equivalent of uniaxial extension called planar extension, and this leads to a planar extensional viscosity as indicated.

But it is now time to get down to specifics. We see a schematic of the experimental apparatus for flow through contractions, and I begin by showing some important results which have been around for some time. They are for the 1% aqueous solution of polyacrylamide I've already referred to, flowing in *planar* contractions. We've already seen that the rheometrical data show a typical shear-thinning response. (SO WE HAVE A SHEAR THINNING LIQUID FLOWING IN A PLANAR CONTRACTION)

I shall show you clear evidence of vortex enhancement as the flow rate is increased in all circumstances, but the mechanism is something we shall need to take note of.

To, first, pictures for a 4:1 planar contraction. The flow rate is increasing as we move through the sequence. There is no doubt that it is the SALIENT corner vortex which grows in the build up to Vortex Enhancement.

Contrast this with what happens in the same liquid in an 8:1 contraction. Here, the fourth picture is a blow up of the previous one to leave is in no doubt that both a lip vortex and a salient corner vortex are now in evidence. The lip vortex encapsulates the salient corner vortex and becomes dominant. (There is again vortex enhancement, but the mechanism is intriguingly different.)

For completeness, I show you that the same mechanism prevails in an 80:1 contraction.

So, for this shear thinning polymer solution, vortex enhancement is found in planar contractions. We shall need to come back to the intriguing side issue of mechanisms, but let's put that on hold for the moment.

I could show you scores of examples where vortex enhancement is found in shear thinning liquids in AXISYMMETRIC contractions. I show you just one for completeness. The liquid is an aqueous solution of polyacrylamide, which we have denoted by C1.

But I now want to pass on to constant-viscosity Boger fluids. I show you first a famous sequence for an axisymmetric contraction obtained by David Boger himself in the 1980s. This has occupied the attention of rheologists, both experimental and computational, for nearly 30 years. Vortex enhancement is clear and dramatic.

So we have vortex enhancement in both planar and axisymmetric contractions for *shear thinning* elastic liquids, and certainly for *axisymmetric contractions,* the same is true for constant viscosity Boger fluids. There is clearly one piece of the jigsaw puzzle that remains. What happens when Boger fluids flow in PLANAR contractions?

Well, no vortex enhancement and that's for sure! In fact, the salient corner vortex is almost swept away in this case, as if inertia were becoming important, which it is not! I show you two further examples of the effect.

Now let's move on to the 'Dynamics'. Let's compare the pressure drop / flow rate response for a Boger fluid and a Newtonian fluid of the same shear viscosity. In the axisymmetric case, the difference is dramatic. Much more pressure is required to produce a given flow rate in the case of the Boger fluid. The Couette correction in this case is huge!

But what do we see in the planar case? Well, virtually NO difference between the responses for the Newtonian and Boger fluids. At the same time, no vortex activity is encountered right up to the onset of instabilities.

So, let me summarize the situation for the four cases.

Now, I've already placed before you enough intrigue to provide material for a cluster of PhD theses. But before I attempt to bring you up to date on the likely causes for the different flow phenomena, let me move on to the flow I mentioned at the beginning, i.e. "splashing".

I shall now need to concentrate on constant-viscosity Boger fluids, but the recipe is such that the viscosities are quite low (of the order of half a Pas). This restriction is important, since the 'window of opportunity' in the splashing experiment is quite limited. If the viscosity is too low, satellite drops can dominate, and, if the viscosity is too high, the flow is innocuous and most uninteresting.

So, the polymer concentrations are now quite low and the test liquids can be considered to be in the 'slightly elastic liquids' category, with very small characteristic relaxation times. However, in the splashing experiments, polymer concentrations as low as 10 wppm can result in dramatic changes in the flow characteristics and in our experiments we have never needed to go beyond 100 wppm.

I have already shown you the various flow features that arise and, in passing, I show you a typical response when a sphere is released to create an axisymmetric Worthington jet. So, let's concentrate on this Worthington jet and how its height can be dramatically affected by a very small amount of viscoelasticity. The time scale of each experiment is very short and the phenomena are often invisible to the naked eye. So, a high-speed camera is very desirable, although I shall concentrate on pictures taken at the maximum height of this jet. Indeed, the actual height of the jet can be estimated very conveniently by simply using a horizontal piece of paper and a ruler.

Sometimes we compare the response of slightly elastic liquids with that of the solvent and sometimes we modify the recipe a little to ensure that the Newtonian and

Boger fluid viscosities are the same. There is no great issue here, since the polymer concentrations we are interested in are so low as to have a trivial effect on the viscosity. We are talking about a couple of % at the most.

The first illustration is one that has been around for some time. It concerns a Newtonian fluid and a Boger fluid of the same shear viscosity. The polymer concentration of 50 wppm means that the characteristic time of the polymer solution is quite low.

Now, the pictures you are looking at tell an obvious story. A slight amount of viscoelasticity can have a huge influence on the height of the jet. Let me quantify the scale of the effect for a given set of conditions, which are fairly typical. We see that concentrations as low as 10 ppm can have a dramatic effect on the height of the jet, although any change in the shear viscosity is insignificant.

In these experiments the type of polymer used is found to be important and the next pictures compare results for a flexible polyacrylamide and a semi rigid xanthan gum.

We clearly need to consider the question of likely mechanisms, but let me first address the same issue that occupied our minds in connection with contraction flows.

The *very* dilute polymer solutions we are investigating in the splashing experiment can be considered to be constant-viscosity Boger fluids, albeit with very low characteristic relaxation times. So the question is:- How will they behave in the two-dimensional *planar* equivalent of the splashing experiment? This involves the release of long horizontal cylindrical rods onto the free surface of the liquid. In passing, I must draw your attention to the quite attractive three-dimensional flow features we encountered when *short* rods were released. They are certainly pleasing on the eye, but they are not particularly relevant to my main theme. So let me now concentrate on the truly planar equivalent of the splashing experiment. This involves the careful release of LONG horizontal rods onto the free surface of a liquid.

The figures show that, in this case, there is essentially *no* change in the maximum height of the jet over the range of conditions that we were able to study. So, shades again of our experience in the contraction flow experiments for Boger fluids. Clearly, the conclusion is independent of the polymer used.

It isn't appropriate here to introduce any substantial shear thinning and I need to press on.

Let us now try to understand and interpret what we have seen so far.

So I next turn to one of my loves, Computational Rheology, and I ask the question: Is it possible to predict and simulate the observed behaviour? To do this, we first need to agree on appropriate constitutive equations for the test liquids.

In the case of constant-viscosity Boger fluids, an immediate choice comes to mind that most would agree to be appropriate and reasonable, at least as a first guess; the so-called Oldroyd B model.

I show you the constitutive equations for this model fluid using a conventional notation and I also show you the associated predictions for some rheometrical flows. Briefly, we have a constant shear viscosity, a positive first normal stress difference N_1, a zero second normal stress difference N_2 and a potentially high extensional viscosity. You will see that this becomes infinite at a finite value of the strain rate, which should be high enough! That is OK for starters.

When we introduce shear thinning into the discussion, the choice seems unlimited! I am simply going to show you numerical simulations for a certain so called Phan-Thien model, which is *relatively* simple and will certainly provide the sought-for shear-thinning.

At the University of Wales Institute of non-Newtonian Fluid Mechanics, we have studied contraction flows employing the two constitutive models I have described and using a variety of numerical methods. I shall show you data from Aberystwyth obtained by my colleague Tim Phillips and from Swansea obtained by my colleague Mike Webster.

I show you first some representative simulations obtained using a finite volume semi-Lagrangian technique for the Oldroyd B model. Both planar and axisymmetric contractions are included. In the axisymmetric case, there is clear evidence of vortex enhancement, while in the planar case this is absent. The simulations are for a constant viscosity model and therefore qualitatively in agreement with the experimental results on Boger fluids.

Let me now pass on to some simulations from my Swansea colleague Mike Webster that I have been associated with. Again they are for the Oldroyd B model, only this time the corners have sometimes been rounded to enable us to reach higher Weissenberg numbers. The simulations confirm the general trend and the vortex *inhibition* in the planar case is now quite clear. So far, so good!

But what about the influence of shear thinning? Well, here are the corresponding simulations for the PTT model, which includes shear thinning. Vortex enhancement is now predicted in both the planar and axisymmetric contractions. So the simulations are in encouraging qualitative agreement with the experimental pictures I showed you earlier.

But let me now ask whether we can also simulate the intriguing mechanisms for vortex enhancement when shear-thinning fluids flow in planar contractions. (You will recall the series of pictures for the shear thinning 1% aqueous solution of polyacrylamide.) Well, let me show you some very recent simulations obtained by a Computational Rheology group from Portugal. They've again used a PTT model to handle shear thinning. So here are their simulations obtained using a finite-volume method; De is a Deborah number.

For the 4:1 contraction we see that the mechanism is clearly associated with the growth of the *salient* corner vortex. However, for a 20:1 contraction, there is a clear evidence of a *lip* vortex, which becomes the dominant influence in generating vortex enhancement. This is even clearer for a 100:1 contract.

Now I find these simulations immensely exciting and encouraging. Quite obviously, the field is making significant progress.

But what of the Couette correction? What of the pressure gradients required to produce a given flow rate in *all* the cases we have discussed. Well, I have to admit to you that, *without exception*, viscoelasticity is predicted to have a negligible effect on the Couette corrections, even when there are substantial changes in the flow structure. Now, we can live with this in the planar contraction flow of constant viscosity liquids, but not for the others. This is very disappointing and worrying and I have to tell you that we are certainly not alone in this very negative conclusion. So far as I am aware, everyone, without exception, has been unable to predict measurable increases in the Couette corrections, and this is certainly one of the unresolved problems in this field.

So, *much* progress but some pessimism also. But let's pass on to 'splashing'. In many ways, this is a more challenging numerical problem, since it is clearly unstable and involves free surfaces.

I have been indirectly associated with some numerical work generated in Sao Carlos in Brazil by Cuminato, Tome and others, ably assisted by Sean McKee of Strathclyde in Scotland. Currently, the simulations are for the simpler falling SPHERICAL DROP problem rather than for a solid sphere, but, although I didn't show it to you, we can expect a similar reduction in the height of the Worthington jet in this case also. So, I simply show you the simulations for the flow development and then for the maximum height of the jet as the non-dimensional number – this time We, the Weissenberg number increases.

Point 1 – They can simulate the flow.

Point 2 – There is a difference in the height of the jet and the trend is in the right direction. So, watch this space!

But I don't want to leave the story in the air, as it were. Why the difference between some planar and axisymmetric flows and why the difference sometimes for constant viscosity and shear thinning fluids. In a hand-waving fashion, we have associated what happens with extensional viscosity phenomena and, in particular, with the high Trouton ratios we know to be present in mobile polymer solutions. I show you that such a view goes back several decades in the case of contraction flows.

But my time is up. All that remains is for me to leave you with a worrying question.

If we are right in believing that we can lay all the extravagant effects at the door of extensional viscosity, why is it that no vortex enhancement is found when Boger fluids flow in planar contractions? So far as I am aware, there is no evidence that Boger fluids only exhibit high Trouton ratios in *uniaxial* extension!

So much progress, but several open questions remain. I find all this immensely exciting and I hope that I have been able to convey some of my enthusiasm to you!