Compression of polymer beads: Finite element modeling of stress-strain behaviour

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ABSTRACT

A finite element method has been used to simulate the deformation of gel beads during uni-axial compression. The simulated data has been used to investigate an experimental method for obtaining an apparent elastic modulus (E_{app}) of single beads. The results indicate that the measured modulus is related to the shear modulus (G) as $E_{app} \approx 4.4G$.

INTRODUCTION

In order to characterize the mechanical properties of polymer gel beads a setup where a single bead is compressed uniaxially has been studied. During compression the applied force, the axial displacement and the central lateral expansion are recorded. Using these data stress-strain curves are derived from which it is desired to extract the elastic shear modulus G. Similar studies have been carried out previously[1,2] using the classical theory of Hertz[3] on force-displacement data. However, for the setup in the present studies it is more desirable to extract the elastic modulus from stress-strain curves. The possibility to do this is investigated here by modeling the compression using a finite element method and comparing simulated data with real measurements.

MODEL DESCRIPTION

The notion used here is that of Bird et al.[4] The model is based on the momentum balance with inertia and gravity neglected. Furthermore, the beads are assumed to be incompressible so that the equation of continuity is satisfied. The stress tensor $\boldsymbol{\tau}$ is split into a purely viscous part and a purely elastic part using the neo-Hookean constitutive equation for the elastic part. The stress tensor is thus given by

$$\boldsymbol{\tau} = -\mu\dot{\boldsymbol{\gamma}} + G\boldsymbol{\gamma}_{[0]} \tag{1}$$

where μ is the viscosity, G is the shear modulus, $\dot{\boldsymbol{\gamma}}$ is the rate of strain tensor and $\boldsymbol{\gamma}_{[0]}$ is the upper convected relative strain tensor. The finite element formulation of the model is obtained by multiplying the momentum balance and the equation of continuity with a trial function ϕ and integrating over the cross-section (S) of a sphere, c.f. Eq. 2 and Eq. 3 (axial symmetry so no changes in θ).

$$\mathbf{0} = \int_S \phi \nabla \cdot (p + \boldsymbol{\tau}) dS \tag{2}$$

$$0 = \int_S \phi(\nabla \cdot v) dS \tag{3}$$

The finite element model is then discretized using linear triangular elements both for velocities and pressures. This method, however, needs an additional term added to the incompressibility condition in order to stabilize the system of equations[5]. The error added by introducing this term is smaller than the discretization error and can thus be made negligible.

RESULTS AND DISCUSSION

The simulation is carried out on a geometry corresponding to a quarter sphere. In Figure 1 a mesh deformed to 70% can be seen. During the simulation the axial force (F) is calculated from the nodal pressures and elastic stresses. The macroscopic strain (γ) is defined as the relative central lateral expansion of the sphere, i.e.

$$\gamma(t) = \frac{R(t)}{R_0} \tag{4}$$

where $R(t)$ is the radius at time t and R_0 is the initial radius. The axial stress at time t acting on the compression interface is defined as the force divided by the area of the central lateral cross section, i.e.

$$\sigma(t) = \frac{F(t)}{\pi R(t)^2} \tag{5}$$

As mentioned above stress-strain data

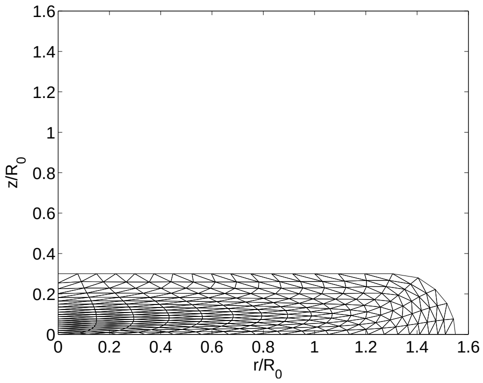

Figure 1: Mesh which has been deformed 70%. Original geometry was the cross-section of a quarter sphere.

from real measurements show an initial linear part from which an apparent modulus is obtained. The simulated curves also show this initial linear dependence and therefore it can be investigated if the apparent modulus is related to the shear modulus G by carrying out simulations with various values of G while keeping μ constant (and small to avoid viscous contributions to the force). These simulations show that the slope of the linear part (E_{app}) scales with the shear modulus approximately as $E_{app} = 4.4G$. This indicates that the apparent modulus is in fact related to the real shear modulus and therefore the experimental method seems to be valid. In Figure 2 a comparison

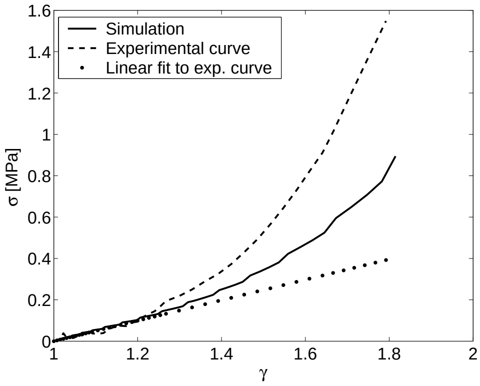

Figure 2: Comparison of a simulated curve with an experimental curve. Also shown is the a linear fit to the experimental curve up to $\gamma = 1.10$. Simulation was carried out with $\mu = 1.2 \cdot 10^4 Pa \cdot s$ and $G = 1.2 \cdot 10^5 Pa$.

of a simulated curve and an experimental curve can be seen. From this figure it is seen that good agreement is observed at low strains $\gamma < 1.22$. However, the model is not able to describe real behaviour at large strains which indicates some degree of strain hardening in the real beads. Furthermore we have noted that it is not possible to simulate purely elastic behaviour, i.e. $R_0 G >> \mu(-\dot{h})$, up to large strains because of instability problems. We are, however, most interested in data at low strains and therefore the instability issues are not critical.

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