

## Critical Review of Drilling Foam Rheology

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### ABSTRACT

In this paper, problems related to the rheology of drilling foams are addressed. Both empirical and analytical foam-modeling approaches are considered. The effects of different fluid parameters on the rheology of foams are thoroughly investigated. A rigorous model comparison is made to evaluate the predictions of the models. Finally, a semi-empirical modeling approach is recommended based on analysis of the analytical and empirical models.

### INTRODUCTION

Foams are being used in a number of petroleum industry applications, which exploit their high viscosity and low density. Foam can be used as a circulation fluid during drilling, well completion and production operations. Drilling foams have the greatest benefits during underbalanced drilling due to their ability to lift large quantities of produced liquids and drilled cuttings. Underbalanced drilling technique allows the drilling of producing zones with minimum formation damage. The low annular velocity that is typical for foam drilling greatly reduces the possibility of borehole erosion. At the same time, a good knowledge of foam rheology and hydraulics is a must for safe and economical drilling.

Drilling foams are complex mixtures of gas and liquid, whose rheological and hydraulic properties are largely influenced by foam quality, liquid phase viscosity,

temperature and pressure. The quality is the volume fraction of gas within the foam. There have been numerous experimental studies<sup>1,2,3,4,5</sup> of drilling foam rheology, covering wide range of quality, liquid viscosity and flow properties. Although there are differences in the results of these studies, the following general conclusions can be drawn: i) rheology of foams mainly depend on the quality and flow rate; ii) surfactant concentration has little effect on foam viscosity at concentration typical of drilling foams; and iii) the foam viscosity increases with increasing liquid phase viscosity.

Beyer *et al.*<sup>1</sup> have found that pressure and temperature influence the foam rheology is mainly by regulating foam quality. Increasing the pressure significantly reduces the volume occupied by the gaseous phase, indirectly reducing the foam quality and viscosity. At constant pressure, increasing the temperature obviously decreases the viscosity of the liquid phase. Consequently, the foam viscosity increases as the temperature decreases. In addition to the rheology, foam quality has great influence on the structure of foam. Figure 1 presents relative viscosity (foam viscosity/liquid phase viscosity) and structural variation as a function of foam quality. Thus, the foam viscosity increases with increasing quality up to about 94%. Thereafter, viscosity moderately decreases with increasing quality, which indicates the start of breaking

down of foam bubbles<sup>6,7</sup>. As the foam quality approaches to 100%, the viscosity drops very rapidly.

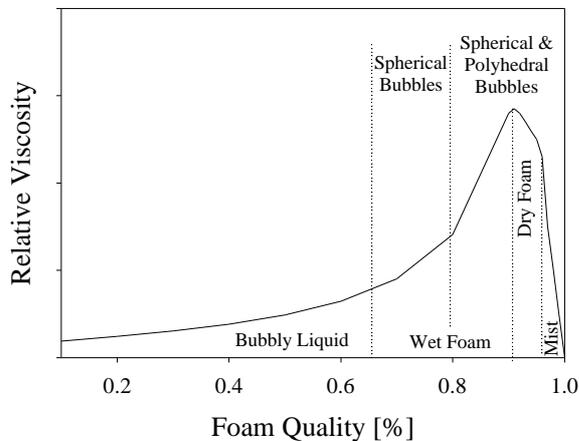


Figure 1 Relative viscosity as a function of foam quality for water based foam

Foam quality is the fundamental descriptive parameter for a macroscopic structure of foams. As we increase the quality of foam from zero to unity, first a bubbly liquid will be formed at low foam quality. However, a critical transition occurs as foam quality increases to rigidity transition. As the term implies, the foam becomes rigid and ordered structurally, and will not flow freely. The structure of foam bubbles appears spherical. This critical value differs slightly depending on the liquid phase compositions. For water-based foams the rigidity transition occurs at about 63% quality<sup>8</sup>.

Further increase in foam quality exhibits peculiar rheological property when the quality exceeds 0.8. The structure of foam bubbles begins to change from spherical to polyhedral configuration. Consequently, the bubbles deform against their neighbors. But they remain separated by thin films of liquid phase that keeps the bubbles from rupture. As the quality approaches 0.946, the bubbles acquire an increasingly polyhedral shape. As a result of this crowding, the foam system attains the highest viscosity close to 0.946 (dry foam limit). Previous studies<sup>9,10</sup> on

foam rheology have found that polyhedral bubbles predominately appear when the foam quality is between 88% and 95%.

Additional increase in the quality of dry foam beyond the dry foam limit moderately decreases the viscosity until foam stability limit is reached. Further increase in quality beyond this limit convert the dry foam into mist, resulting a drastic viscosity reduction. The stability limit of water-based foam is about 97.5%. Addition of viscosifiers into the liquid phase increases the stability limit<sup>6</sup>.

Foam rheological studies typically adopt one of several distinct approaches. A purely empirical approach involves formulating a constitutive law for a material on the basis of experimental data alone. Such laws can be very useful as they are based on experimental observation of real systems. Nevertheless, the form of the equation generally has no physical basis. An alternative approach is to produce a mathematical model in which explicit account is taken for the foam structure and flow properties of each phase to determine the flow properties of foams. However, problems frequently arise in such theories because of assumptions and idealizations that are necessary to simplify the mathematics of the real problem. As a result, mathematical models often bear little resemblance to the physical problem. Consequently, a semi-empirical approach, which has a physical basis but involves experimentally determined constants, is often preferred.

## EMPIRICAL FOAM MODELS

Underbalanced drilling can be performed using different types of foams. Often foam drilling are classified as stable foam (water based foam) or stiff foam (polymer foam) drilling. In stable foams the liquid phase can contain surfactants, salts and corrosion inhibitors, none of which has a significant impact on the viscosity of the liquid phase. Stiff foams contain viscosifiers in addition to these additives<sup>6</sup>.

### Stable Foams

After measuring foam rheology using small diameter tubes, Mitchell<sup>4</sup> found that the foam viscosity  $\eta_f$  is related to its quality and the viscosity of the liquid phase by:

$$h_f = h_L(1 + 3.6\Gamma) \quad (1)$$

where  $\Gamma$  and  $\eta_L$  denote the foam quality and liquid phase viscosity respectively. This relation is applicable when  $\Gamma \leq 0.54$ . For  $\Gamma \geq 0.55$ , the study suggested another correlation, which is given by:

$$h_f = h_L(1 - \Gamma^{0.49})^{-1} \quad (2)$$

Sanghani and Ikoku<sup>7</sup> experimentally studied foam rheology with a concentric annular viscometer that closely simulated actual borehole conditions. They concluded that foam is a power-law fluid with flow behavior index  $n$  and flow consistency  $K$ , which are both functions of foam quality. Recently, Martins *et al.*<sup>11</sup> have determined the relationship between these parameters and the foam quality as:

$$n = a_1 \left( \frac{1 - \Gamma}{\Gamma} \right)^{a_2} \quad (3)$$

$$K = b_1 \left( \frac{1 - \Gamma}{\Gamma} \right)^{b_2} \quad (4)$$

where  $a_1$ ,  $a_2$ ,  $b_1$  and  $b_2$  are empirical constants whose values are determined by foam bubble size and liquid phase properties.

### Stiff Foams

Stiff foams usually made by adding a foaming agent to a fairly thin, unweighted drilling mud. Essentially the same structures are seen in both stiff and stable foams. However, stiff foams can be created with a higher quality than the stable foams. The

rheology of stiff foam greatly depends on the liquid phase viscosity and the foam quality. Reidenbach *et al.*<sup>2</sup> investigated the flow properties of foams at different qualities and viscosifying polymer concentrations. Results indicated that the rheological behavior of foam fluid is primarily that of a yield power-law fluid and can best be described by a Herschel-Bulkley model. Accordingly, the yield stress,  $\tau_y$  is given by:

$$t_y = a\Gamma, \quad \text{for } \Gamma \leq 0.6 \quad (5)$$

and

$$t_y = be^{d\Gamma}, \quad \text{for } \Gamma \geq 0.6 \quad (6)$$

where  $\alpha$ ,  $\beta$  and  $\delta$  are empirical constants that vary with liquid and gas phase properties. The foam consistency coefficient and the flow behavior index are represented by  $K_f = K_L$  and  $n = \alpha\Gamma + \beta\Gamma^2$ . Cawiezel and Niles<sup>3</sup> investigated the rheological properties of foams at downhole conditions. The result confirmed that the yield power-law model best describes the rheology of foams. However, a problem with this approach is that the empirical parameters need to be determined exactly for different foam systems encountered in a drilling operation. Kraynik<sup>12</sup> has suggested that empirical correlations based on foam-flow data have limited predictive value when the connection between structure and rheology is lacking. Therefore, another method of foam rheology prediction is necessary.

Valkó and Economides<sup>13</sup> have presented principle of volume equalization to describe the rheology of foams. The technique uses the specific-volume expansion ratio,  $\epsilon$  as the additional parameter representing the phase relation of the gas and liquid. This quantity is defined as the ratio of the liquid density to the foam density, which varies along the flow path because of the change in the

pressure. Thus, the specific-volume expansion ratio is given by:

$$\mathbf{e} = \frac{\mathbf{r}_L}{\mathbf{r}_f} \quad (7)$$

The principle of volume equalization is derived from an invariance requirement. It assumes that for a straight duct flow of constant cross section, both compressible and incompressible flows possess the invariance property. This means that the loss of mechanical energy is proportional to the kinetic energy, in other words the Reynolds number is constant<sup>14</sup>. If we demand the same invariance property for the flow of a compressible non-Newtonian fluid, this restricts the form of the constitutive rheological equation. Constitutive equations, providing the required invariance, are called volume equalized. The volume equalized power law equation is given by<sup>14</sup>:

$$\frac{\mathbf{t}}{\mathbf{e}} = K \left( \frac{\mathbf{g}}{\mathbf{e}} \right)^n \quad (8)$$

One of the advantages of this approach is that the volume equalized wall shear stress versus the volume equalized equivalent Newtonian shear rate plotted on a log-log scale results in a straight line for a given gas-liquid pair for a wide range of foam qualities and pressures.

Recently, Ozbayoglu *et al.*<sup>5</sup> have performed a comparative study to investigate the predictive ability of the available foam hydraulic models. They have conducted flow experiments using stable foam at 70, 80 and 90 % foam qualities. The result illustrated that the power law model can better characterize the rheology of foams at 70 and 80 % qualities, whereas the rheology of foam at 90 % quality best fits with the Bingham plastic model. Comparison of the model predictions with experimental pressure loss values indicated that model predictions of frictional pressure losses can be significantly different from the

actual values. In spite of this, Valkó and Economides<sup>13</sup> model gives relatively accurate pressure loss predictions.

## ANALYTICAL FOAM MODELS

Mathematical modeling of foam rheology is a challenging task. Since microscopic structure and dynamics of foams are poorly understood and remain a subject of basic scientific interest to physicists and engineers. Often foam flows are characterized by bubble deformations and rupturing that have significant effect on the rheological properties of foams. In steady homogenous shear flow, small change in bubble shape from sphericity depends upon a capillary number  $Ca$ , which is given by:

$$Ca = \frac{a\mathbf{g}h_f}{\mathbf{s}} \quad (9)$$

where  $a$ ,  $\sigma$  and  $\gamma$  denote average bubble radius, interfacial tension and shear rate respectively. The capillary number is a relative measure of viscous forces that tend to distort the bubble and interfacial tension, which favors sphericity.

Hatschek<sup>15</sup> attempted to relate concentrated emulsion viscosity to its structure by  $\eta_f = \eta_L (1 - \Gamma^{0.33})^{-1}$ . The formula has been applied to high quality foams at low  $Ca$ . Sibree<sup>16</sup> experimentally measured viscosity of foam at high  $Ca$  and observed similar quantitative dependency upon quality. The liquid phase in his experiments was strongly non-Newtonian and the foam quality was in the range of 0.52 to 0.73<sup>17</sup>.

Recently, Llewellyn *et al.*<sup>17</sup> have studied the rheology of bubbly liquids at low  $Ca$  and developed a semi-empirical model for  $\Gamma \leq 0.5$ , which is based on theoretical analysis of Frankel and Acrivos<sup>18</sup>. Accordingly, for steady flow with small bubble deformations (i.e. small  $Ca$ ) the foam viscosity is given by:

$$\mathbf{h}_f = \mathbf{h}_L(1 + 9\Gamma) \quad (10)$$

The model predictions show good agreement with experimental observations.

Based on a mathematical treatment of the behavior of liquid droplets with an elastic bounding membrane, suspended in a viscous liquid, Barthes-Biesel and Chhim<sup>19</sup> developed a constitutive equation for dilute suspensions that account for high values of Ca as:

$$\frac{\mathbf{h}_f}{\mathbf{h}_L} = 1 + (2.5 - \psi Ca^2)\Gamma \quad (11)$$

where  $\psi \approx 68.5 + 21\phi + 60\lambda$ . The constant  $\phi$  depends on non-linearity of the membrane material. Viscosity ratio  $\lambda$  is defined as the ratio of the viscosity of dispersed phase to that of continuous phase. Since the surface tension is constant, the constant  $\phi$  is negligible for foam bubbles. The viscosity ratio for dry foam ranges from 0.02 to 0.03. Therefore, reasonable value of  $\psi$  is about 70. For  $Ca \geq (2.5/\psi)^{0.5}$  the viscosity of the foam will be less than the liquid phase viscosity. Moreover, for very small capillary number ( $Ca \ll 1$ ), Eq. 11 becomes the Einstein's equation for dilute suspension of solid particles, where  $\Gamma$  is solid particle volume fraction.

Currently, widely cited foam modeling studies<sup>20,21</sup> are those of Khan and Armstrong. In these studies, a two-dimensional foam model has been developed for dry foams (i.e.  $\Gamma \geq 0.946$ ) by assuming hexagonal and monodispersed foam cell structure. A general expression for the stress is obtained, which gives the relative viscosity in terms of  $\Gamma$  and Ca. After adoption and simplification, the Khan and Armstrong<sup>20</sup> expression reads:

$$\frac{\mathbf{h}_f}{\mathbf{h}_L} = \frac{0.31}{Ca} + 26\sqrt{\Gamma}(1 - \sqrt{\Gamma}) \quad (12)$$

Thus, for Newtonian base fluid, the magnitude of the yield stress is directly proportional to the liquid surface tension and inversely proportional to the bubble size. Since  $Ca = a\gamma/\sigma$ . Harris<sup>22</sup> has measured foam bubble diameter at different pressure and found that the size of the bubble diameters ranges from 10  $\mu\text{m}$  to 2.4 mm, while the mean diameter varies with pressure from 800  $\mu\text{m}$  at 6.5 MPa to 1.9 mm at 0.5 MPa. Plotting the result on pressure versus bubble volume graph indicates that the bubble size is correlated with the pressure according to the ideal gas equation. Therefore, the bubble size can be predicted for the actual borehole conditions if it is known at the surface. Bubble size at the surface mainly depends on foam production process and the properties of the liquid phase<sup>23</sup>.

According to equation 12, the interfacial tension is another parameter that affects the yield stress of foams. Increasing temperature. Surface tension of ordinary water substance linearly decreases with temperature<sup>23</sup>. In addition to the temperature, the concentration of surfactant greatly affects the surface tension of water. This reduction in surface tension is normally seen below the critical micelles concentration (the concentration at which micelle formation becomes significant). Above the critical micelles concentration, the influence of concentration on surface tension becomes insignificant<sup>24</sup>.

Princen and Kiss<sup>25</sup> presented a semi-empirical foam model, which is applicable for high quality foams (i.e.  $0.73 \leq \Gamma \leq 0.98$ ) at low capillary number ( $10^{-4} \leq Ca$ ). The model is based on theoretical analysis of foam microstructure and experimental investigation of rheology of emulsions. Princen and Kiss<sup>25</sup> defined the yield stress as:

$$\mathbf{t}_y = \frac{\mathbf{s}\Gamma^{0.33}}{a} F(\Gamma) \quad (13)$$

where the function  $F$  can be approximated by  $F(\Gamma) = 0.080 - 0.114 \log(1 - \Gamma)$ . Thus, the relative viscosity is given as:

$$\frac{h_f}{h_L} = \frac{F(\Gamma)\Gamma^{1/3}}{Ca} + 32 \frac{\Gamma - 0.73}{Ca^{1/2}} \quad (14)$$

### SLIP AT THE WALL

Slip at the wall is another interesting and important characteristic of foam flows. A convenient macro scale description of wall slip mechanism is based on the existence of a thin fluid layer that does not itself slip but wets the wall and lubricates the foam flow. Experimental evidences for tube and rotational flows indicates that the wall slip occurs above finite values of wall shear stress called yield slip stress,  $\tau_{sy}$ . Because of this, various flow behavior of the foams have been recorded, such as shear flow, plug flow and slip flow as shown in Fig. 2<sup>12</sup>.

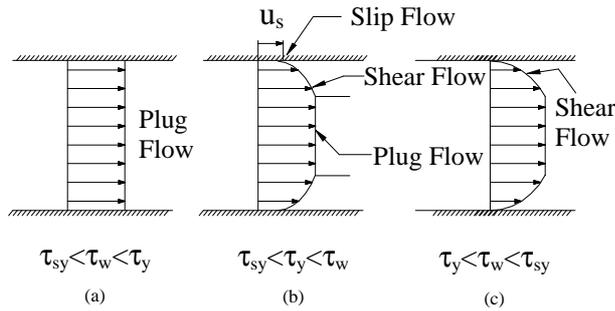


Figure 2 Velocity Profiles in Foam Flows.

In figure 2a we have a plug flow with uniform velocity profile. This type of flow occurs when the wall shear stress,  $\tau_w$  is greater than the slip yield stress and less than the yield stress. Beyer *et al.*<sup>1</sup> reported the existence of plug flows during foam flow experiment. However, when  $\tau_{sy} < \tau_y < \tau_w$ , we have slipping Bingham fluid flow with slipping velocity,  $u_s$  as shown in Fig. 2b. Normal Bingham fluid flow with no-slip condition at the wall is shown in Fig. 2c, which occurs when  $\tau_y < \tau_w < \tau_{sy}$ . Thondavadi and Lemlich<sup>26</sup> investigated flow properties

of foam with and without solid particles. But they did not observe slip in rough pipes, whereas it is commonly occurring in smooth ones. For flow in smooth pipe, the slip velocity for dry foam is estimated by<sup>12</sup>:

$$u_s = \frac{Ca_s s}{h_L} \quad (15)$$

where  $Ca_s$  is modified capillary number given by<sup>27</sup>:

$$Ca_s^{1/3} = \frac{6at_w \sqrt{1 - \Gamma}}{s(1 - 3.28\sqrt{1 - \Gamma})} \quad (16)$$

Like the ordinary capillary number, the modified capillary number is a relative measure of viscous forces at the wall that tend to distort the foam bubble and interfacial tension, which favors sphericity. The modified capillary number becomes negative for foam quality less than 0.91. Since Eq. 15 is applicable for dry foam when  $Ca_s \ll 1$ . Slip velocities of water based dry foams with surface tension of 0.022 N/m and bubble radius of 1 mm are estimated using the model and presented in Fig. 3. The figure shows that the slip

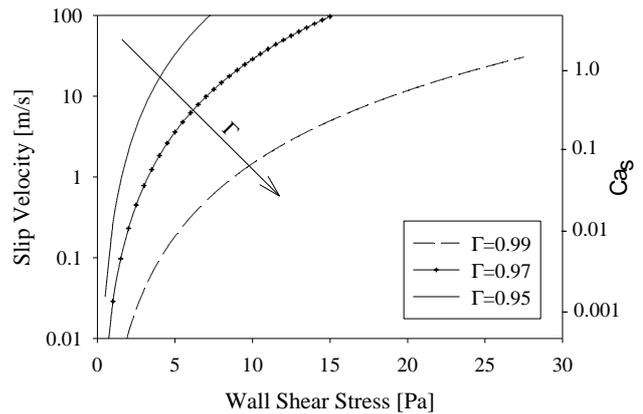


Figure 3 Slip Velocities of Dry Foams as a Function of Wall Shear Stress.

velocity decreases as the foam quality increases. This means that in dry foam

range, high quality foams require higher wall shear stress than low quality foams to have the same slip velocity. It is apparent from the figure that the slip velocity significantly increases with increasing the wall shear stress. Figure 3 is in qualitative agreement with the observations of Beyer *et al.*<sup>1</sup>, who also found that slip velocity increases with liquid volume fraction of foam. However, the model overestimates the slip velocity when  $Ca_s$  is greater than 0.1.

Practical importance of the slip velocity is to find accurate relationship between the flow rate and the pressure drop. As seen from Fig. 2, the foam flows experience different types of velocity profiles. If we consider laminar flow of power law fluids and assume a differential length of pipe throughout which the density may be considered constant, then mean flow velocity  $U$  in a circular pipe can be expressed as<sup>27</sup>:

$$U = u_s + \sqrt{-\frac{dp}{dx} \frac{D Re_n}{32 r_f}} \quad (17)$$

where  $dp/dx$  denotes pressure gradient and  $Re_n$  is generalized Reynolds number given by:

$$Re_n = \frac{r_f (U - u_s)^{2-n} D^n 2^{3-n}}{K(3+1/n)^n} \quad (18)$$

where  $D$  is diameter of the pipe.

#### MODEL COMPARISON

So far we have seen different foam models based on analytical and empirical observations, but it is very important to select the most appropriate foam models that can be used to predict flow behaviors of drilling foams. Figure 4 presents relative viscosity profiles of foams based on the Hatschek<sup>15</sup> and Llewellyn *et al.*<sup>17</sup> models. The two graphs follow the same pattern for  $\Gamma \leq 0.6$  with maximum difference of about

20%. It is apparent from the figure that the two graphs cross at 60% foam quality, which is very close to the rigidity transition limit. Thus, the Hatschek<sup>15</sup> model, which is recommended for  $\Gamma \geq 0.52$  shows significant increase in the relative viscosity. This can be attributed to the structural change occurs at the rigidity limit. Therefore, the semi-empirical model of Llewellyn *et al.*<sup>17</sup> is more reliable than other models for bubbly liquid ( $\Gamma \leq 0.6$ ) at low  $Ca$ . The classical model of Hatschek<sup>15</sup> is still applicable for bubble liquids and foams ( $0.0 \leq \Gamma \leq 0.8$ ) at low  $Ca$ .

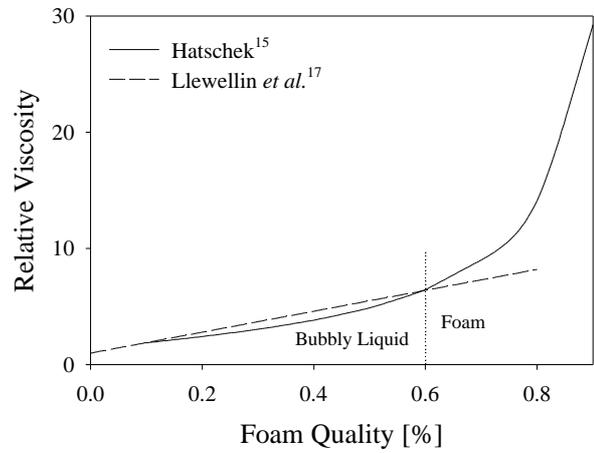


Figure 4 Relative Viscosities of Foams Using Hatschek<sup>15</sup> and Llewellyn *et al.*<sup>17</sup> Models.

In the upper part of a well, drilling foams can be in the “dry foam” limit, where the bubble shape is dominantly polygonal. Although several analytical dry foam models have been proposed, their usages are very limited. Since they are very complicated and inconvenient for practical use. Most widely accepted dry foam model is that of Khan and Armstrong<sup>20</sup>. However, this model still suffers several restrictions and idealizations. Therefore, it requires calibration to account for the limiting factors. For high quality foams Eq. 12 can be rewritten in a simple form as:

$$\frac{h_f}{h_L} = \frac{0.31}{Ca} + 13(1 - \Gamma) \quad (19)$$

Thus, the dry foam plastic viscosity ( $13\eta_L(1 - \Gamma)$ ) decreases with increasing the foam quality. This is in agreement with previous experimental observations, which is commonly seen when foam quality is over 94.6%. Previous studies<sup>6,7</sup> on dry foam rheology suggested that the viscosity moderately decreases with increasing quality up to the foam stability limit, which is 97.5% for water based dry foams. At low Ca ( $Ca < 0.001$ ), the second term in the right hand side of Eq. 19 becomes negligible. This contradicts with the experimental observations<sup>7</sup> that found moderate decrease in viscosity as the quality of dry foam increases.

Figure 5 compares the yield stress predictions of Khan and Armstrong<sup>20</sup> and Princen and Kiss<sup>25</sup> model as a function foam quality. The ratio of yield stresses is defined as the ratio of predictions of Khan and Armstrong model to the predictions of Princen and Kiss<sup>25</sup> model. As seen from the figure, the predictions of Khan and Armstrong model are significantly higher than that of Princen and Kiss<sup>25</sup> model. However, very close to unity the predicted values are comparable, reaffirming again the restrictions of Khan and Armstrong model (i.e. applicable only for dry foams).

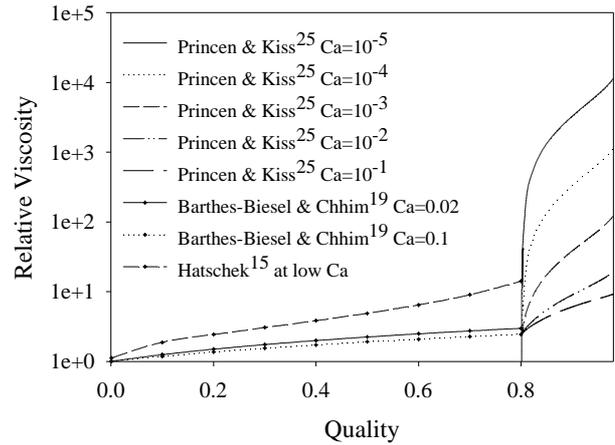
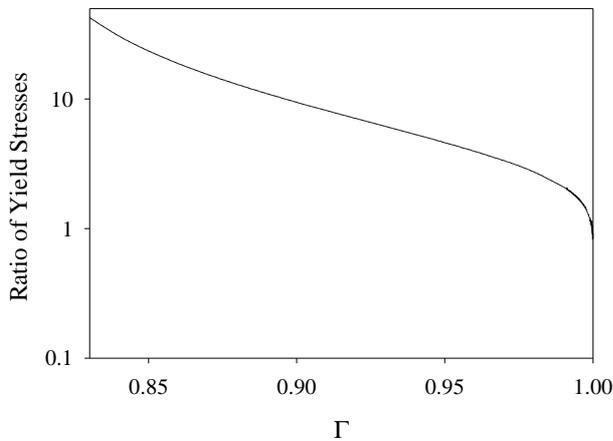


Figure 5 Yield Stress Ratio Versus Foam Quality.

Figure 6 presents relative viscosity predictions of foam models as a function of quality and capillary number. The figure shows that the influence of quality on relative viscosity is considerably high at high capillary number. It is also interesting to note that relative viscosity predictions of

Figure 6 Relative Viscosity Predictions of Foam Models.

Brathes-Biesel and Chhim<sup>19</sup>, and Princen and Kiss<sup>25</sup> models are the same when foam quality is about 0.8. Moreover, the patterns of curves apparently indicate a sharp increase in relative viscosity, which is commonly seen during foam rheology measurement. Therefore, smooth relative viscosity predictions can be obtained if Brathes-Biesel and Chhim<sup>19</sup>, and Princen and Kiss<sup>25</sup> models are employed for  $0.0 \leq \Gamma \leq 0.80$  and for  $0.8 \leq \Gamma \leq 0.95$  respectively. As the capillary number decreases, the relative viscosity curves of Brathes-Biesel and Chhim<sup>19</sup> move up, approaching the Hatschek<sup>15</sup> model curve, which is often applied at low Ca.

## CONCLUSION

- The rheology of drilling foams can be estimated using different semi-empirical

models, which take account for both foam quality and capillary number. These two foam flow parameters can successfully describe the rheology of drilling foams.

- Purely empirical foam rheological corrections are unreliable beyond the original data upon, which they are based. However, simplified analytical models can provide valuable physical insight to develop more reliable semi-empirical models.
- Flow and rheological properties of drilling foams greatly change as the foam quality increases. Therefore, obtaining a single foam rheological model can be practically difficult.
- Wall slip, and bubble interference, deformation and rupturing are fundamental characteristics of foam flow, which affect both rheology and hydrodynamic.

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