Gas-assisted Displacement of Non-Newtonian Fluids

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ABSTRACT

The gas-assisted displacement of a Oldroyd-B viscoelastic fluid contained in a circular tube (with inner radius R_0) is modelled numerically. Good agreement between the simulations of the developed gas channel and the displacement experiments performed by Hyzyak and Koelling[1] is obtained, comparing fractional coverage (defined as $1-R/R_0$ where R is the radius of the penetrating gas front).

INTRODUCTION

During the resent years several publications (for instance Hyzyak and Koelling[1] and Gauri and Koelling[2]) have concerned gas assisted displacement of viscoelastic fluids (polymer melts and polymeric solutions) contained in a circular cylinder: As the gas is injected into one of the ends of the cylinder, it leaves a fluid layer of uniform thickness on the inner surface of the cylinder. This is a simple model system used to investigate the gas-fluid displacement, as the problem is reduced to an axis-symmetric flow problem.

The understanding of this gas-fluid displacement process is relevant for the geometrically much more complex polymer processing operation Gas-assisted injection moulding (GAIM). This is a process, where a mould is filled partly with a molten polymer, followed by the injection of an inert gas into the core of the polymer melt.

The numerical analysis of the fluid flow, concerning the gas fluid displacement, have to our knowledge, only been based on Newtonian or generalised Newtonian fluid models until now. As polymer melts and polymeric solutions are viscoelastic fluids an increased understanding of the displacement process can be achieved performing numerical simulation based on a viscoelastic model. This is especially important in processes that are dominated by stretch (e.g. elongation) of the fluid, as the GAIM. The stretch occurs in the fluid being displaced in front of the gas.

Here we will focus on the work by Hyzyak and Koelling[1]. They performed displacement experiments on diluted solutions of linear polymers, normally referred to as Booger fluids. These fluids have almost constant shear viscosities and elongational viscosities several order of magnitudes larger than the shear viscosities, at elongational rates above a certain value.

RHEOLOGY

The simplest possible model to describe the constitutive equation of Boger fluids is the Oldroyd-B model. This model has, with success, been able to describe the complex flow behaviours of Boger fluids. Though, refinements in the flow analysis can be obtained using more complex constitutive models. To keep the flow analysis as simple as possible the Oldroyd-B constitutive model will be used throughout this paper.

The stress build-up in the fluid, e.g. the Oldroyd-B constitutive equation, is given in integral form as[3]

$$\mathbf{t} = -\frac{b}{1-b} h_P \dot{g} + \int_{-\infty}^{t} \frac{h_P}{l^2} \exp\left(-\frac{t-t'}{l}\right) g_{[0]}(t,t')\,dt' \qquad (1)$$

The stress comes from two contributions: The polymer stress given by a memory integral (the exponential function) over the finite strain tensor, $g_{[0]}$, and the solvent contribution as a Newtonian term. Note that ß is a non-dimensional parameter, where 0<ß<1.

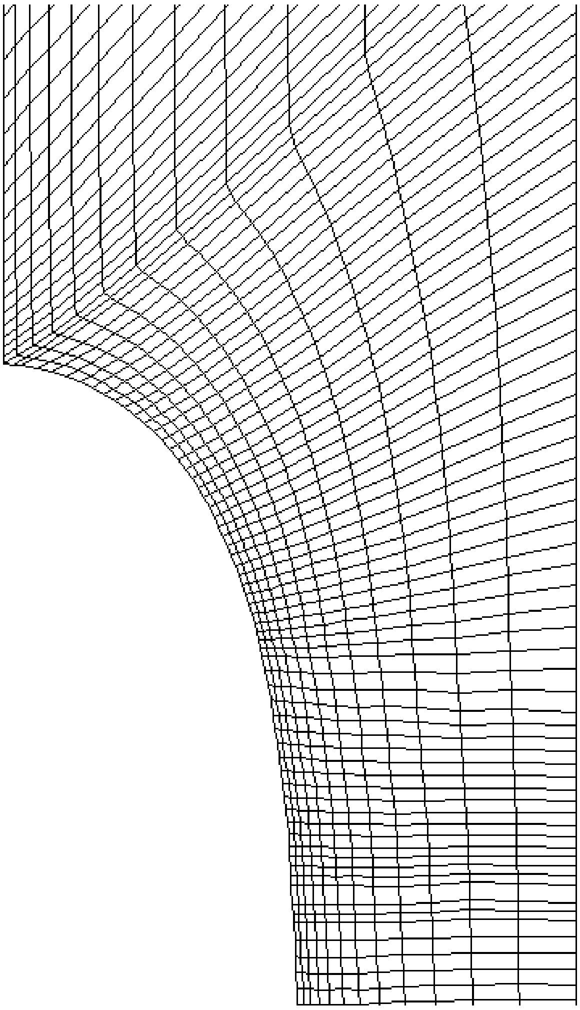

Figure 1. Finite element mesh in the area around the flow front, in a cylindrical coordinate system, (r,z).

NUMERICAL MODELLING

A numerical method is needed in order to calculate the flow of the viscoelastic fluid during the displacement. To model the displacement numerically, the time-dependent finite element method from Rasmussen[4] is used. This method has second order convergence both in time and spatial discretization.

An example of a part of an actual finite element mesh at the end of one of the simulations is illustrated in Fig. 1. The boundary conditions used in the (axis-symmetric) simulations are no-slip conditions on the cylinder wall, no shear stresses and constant normal stress plus pressure on the free surface where the gas is displacing the fluid and an imposed Newtonian velocity profile at the outlet.

NON-DIMENSIONAL GROUPS

The important non-dimensional groups in the displacement are the Deborah number, the surface elasticity, the Capillary number and of course the viscosity ratio ß from the Oldroyd-B model. The Deborah number is in a general definition (e.g. independent of constitutive equation) given as the ratio between the first normal stress coefficient, $?_1$, and the total viscosity, ?, (this ratio represent a characteristic time constant of the process) and multiplied with a characteristic deformation rate in the process (here the steady shear rate at the wall of the tube). Hence

$$De = \frac{y_1(\dot{g}_W)}{2h(\dot{g}_W)}\dot{g}_W \qquad (2)$$

In this form all parameters depends on the characteristic deformation rate of the process. For the Oldroyd-B constitutive equation

$$De = (1-b)\,l\dot{g}_W = 4(1-b)\,l \cdot U/R_0 \qquad (3)$$

where U and R_0 are the average velocity of the flow and the radius of the cylinder, respectively.

The classical non-dimensional measure of the surface tension is the Capillary number, Ca, given as the ratio of the viscous stresses relative to the surface tension (s) stresses. Note that in all our following calculations we use Ca=0 (no surface tension, s) as Hyzyak and Koelling[1] scale out the effect of the surface tension in the performed experiment in a way discussed later.

Another non-dimensional measure of the surface tension is the surface elasticity number. It is given as the ratio of the surface tension stresses relative to the elastic modulus. This type of measure is only material dependent. For the Oldroyd-B constitutive model it is normally defined as

$$f = \frac{s}{R_0}\frac{h_p}{l} \tag{4}$$

Note that the Newtonian viscous part of the constitutive equation is not included in this parameter.

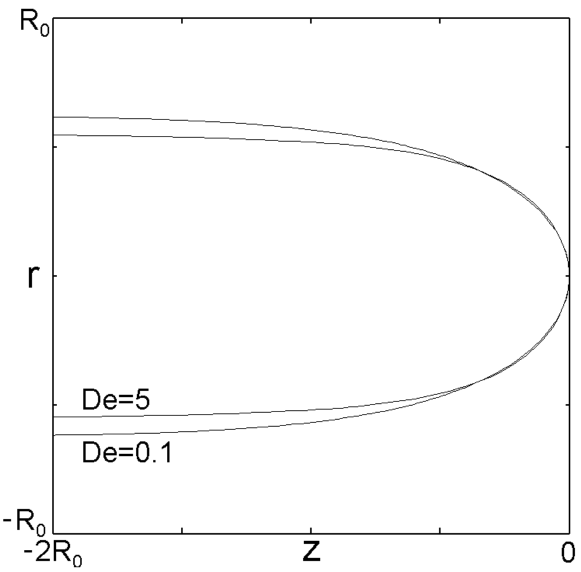

Figure 2. Shape of the flow front in a cylindrical coordinate system, (r,z). The simulations at De=5 (inner curve) and De=0.1 (outer curve) at steady state.

FLOW FRONT

The numerically calculated steady shape of the flow front is shown in Fig. 2 at high and low De number. Note that the axes are in scale.

The flow front is, at high De numbers, suppressed due to the bi-axial elongational deformation in the area around the tip of the flow front, developing strong elastic forces in the fluid.

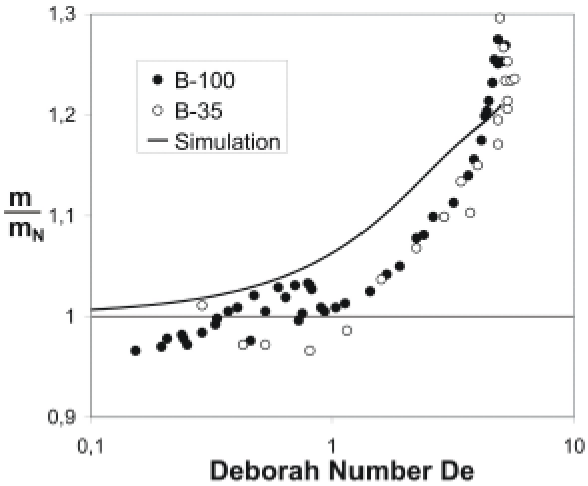

Figure 3. Reduced fractional coverage $\frac{m}{m_N}$ as function of the De number. The line represents Oldroyd-B simulations (using ß=0.5), and the ? (fluid B-35) and ? (fluid B-100) are the experiments from Figure 8 in Hyzyak and Koelling[1].

THE FRACTIONAL COVERAGE

Using the above definitions good agreement between the Oldroyd-B displacement simulations and the experiments by Hyzyak and Koelling[1], is obtained, comparing the fractional coverage, m. The fractional coverage is defined as $m=1-R/R_0$ where R is the radius of the penetrating gas front.

The fractional coverage is a function of the non-dimensional parameters (ß, De and Ca), though Hyzyak and Koelling[1] scale out the effect of the surface tension, dividing the fractional coverage with the fractional coverage from a Newtonian displacement experiments, m_N, with the same Capillary number, leaving out the effect of ß and De.

In the calculations we use a ß value of 0.5, which correspond to average value of the two fluids used in the experiments by Hyzyak and Koelling[1]. Further experimental details and information of the properties of the two fluids (named B-35 and B-100) may be found in Hyzyak and Koelling[1].

The steady-state reduced fractional coverage as a function of the De number is shown in Fig. 3. The line is the numerical simulations while the individual symbols are the experiments from Hyzyak and Koelling[1]. The experiments and simulations show the same qualitative dependence on the Deborah number, indicating that the Oldroyd-B model gives a reasonable prediction of the steady fractional coverage.

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