

On the Rheological Mechanisms Governing Drill-in Fluid Invasion into Reservoir Rocks

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ABSTRACT

This article deals with the comprehension of the rheological mechanisms relevant to *drill in* fluid invasion through reservoir rocks, aiming the design of a solids free non-invasive fluid. Experimental results indicate that highly concentrated solution presented relevant deviations to Darcy's Law. Expressions for resistive force estimation as functions of the first normal stress difference, linear viscoelastic parameters and Trouton ratios are proposed to account for the extra friction losses.

INTRODUCTION

The role of rheology on drilling fluid invasion in the oil reservoir is still not clearly stated. Main points which arise are: to what nature of efforts the fluid is submitted when flowing through the porous media? Which shear rates characterize the well-reservoir boundary? Which rheological properties govern the invasion phenomenon?

Designing *drill in* fluids, which can guarantee minimum invasion into the reservoir rock is a must for open hole completion wells. The industry has proposed several ideas to deal with the problem, most of them based on adding bridging agents to the fluid formulation. Such agents would block pores near the well bore and, consequently, prevent additional fluid to invade the rock.

Several authors (Audibert et al.¹ and Navarrete et al.², etc.) present relevant theoretical and experimental studies on the filtration properties of water based fluids.

Those tests were run both in dynamic and static conditions for several types of fluid, pH, solids size, shape and concentration, pressure and shear stresses. Lomba et al.³ introduced a discussion on additional mechanisms, besides bridging, which could minimize fluid invasion. Among them, there is a topic on how polymers of different rheological properties would behave while flowing through a non consolidated sand bed in a static filtration apparatus specially designed to evaluate invasion. The authors concluded that shear viscosity was not the only factor, which governs invasion, since some less viscous fluids presented less invasive behavior, for the same conditions, than high viscosity fluids. Martins et al.⁴ present a new series of experiments where viscous effects were not enough to predict filtration profiles of polymeric solutions through consolidated porous media. A correlation with normal stress differences is proposed. The authors postulate two additional hypotheses on the role of the viscoelastic properties of such fluids on the invasion behavior. The purpose of the present article is to quantify such hypotheses and establish the advantages and disadvantages of each of them.

BACKGROUND

Consider a static filtration experiment, where a non Newtonian fluid, when submitted to a constant pressure differential, flows through a porous medium previously saturated with the same fluid. Neglecting hydrostatic effects, the collected volume (V) along the

time (t) can be predicted, according to Darcy's Law by the following expression:

$$V = \frac{\Delta P \cdot A \cdot K}{m_{ef} \cdot L} \cdot t \quad (1)$$

Where ΔP is the imposed pressure differential, A and L are the area and thickness of the porous medium and m_{ef} is the effective viscosity of the fluid. Several researchers (Smit and du Plessis⁵, Siskovic et al.⁶, Massarani and Silva Telles⁷ among others) present experimental results confirming the adequacy of Darcy's Law to reproduce the flow of Newtonian and non - Newtonian fluids through porous media, till a certain range of superficial velocities. For higher velocities (typical of gas or high differential pressure flows) additional inertial resistive terms should be considered (Forchheimer Law).

The effective viscosity of a non - Newtonian fluid varies with the shear rate and can be estimated by the ratio between shear stress and shear rate at each point. The shear rate in the porous medium, (i), can be estimated, as a function of the superficial velocity (q) by :

$$\dot{\gamma}^* = \frac{q}{\sqrt{K}} \quad (2)$$

The analysis of viscoelastic effects of fluids flowing through porous media, however, is not a new subject. Several authors, from different areas including fundamental rheology, EOR and drilling/completions have made qualitative observations about the fact. The more relevant works are highlighted in the next paragraphs.

Data presented by Dauben and Menzie⁸, Cakl et al.⁹ and Marshall e Metzner¹⁰ indicate higher friction losses than the ones predicted by Darcy's Law. The authors attribute the fact to normal stresses effects, typical of viscoelastic flows.

Durst et al.¹¹ propose that the total strain experimented by non - Newtonian fluid

flowing through porous media is a composition of shear and elongation efforts.

Jones and Walters¹² introduce the importance of the extensional components in polymer injection through porous media, aiming EOR applications. Saasen et al.¹³ correlated fluid invasion with linear viscoelastic parameters. Later, Svendsen et al.¹⁴ show results of extensional viscosity as indicators of fluid invasion governing parameters. Young et al.¹⁵ show similar correlation for fracturing fluid invasion. These authors suggest that the Trouton Ratio (a relation between extensional and shear viscosity at similar shear/extension rates) would be the major rheological parameter governing fluid invasion.

Bird et al.¹⁶ describe in detail the several material functions which can characterize the viscoelastic steady and unsteady flow of fluids in several geometries. Among them, the following flows and material functions, obtained in simple shear flows are considered in this study:

a) Steady shear flows

$$t_{21} = m_{ef} I_{21} \quad (3)$$

$$t_{11} - t_{22} = N_1 \quad (4)$$

$$t_{22} - t_{33} = N_2 \quad (5)$$

Where N_1 and N_2 are known as the first and second normal stress differences, respectively. t_{21} and $\dot{\gamma}_{21}$ represents the shear stress and shear rate, while t_{11} , t_{22} and t_{33} are the normal stress components. The measurement of the first normal stress difference can be made in commercial shear flow rheometers, although there is a lot of controversy among the manufacturers on the most reliable method to do it. There is no commercial equipment available for the measurement of the second normal stress differences, although a few research efforts are reported (Keentok et al.¹⁷, Christiansen and Leppard¹⁸). These authors state that N_2 is in the order of magnitude of 10 to 15 % of N_1 .

For a Newtonian fluid, N_1 and N_2 are equal to zero.

b) Small amplitude Oscillatory Shear Flows

$$\mathbf{t} = G' \mathbf{g}_0 \sin \omega t + G'' \mathbf{g}_0 \cos \omega t \quad (6)$$

where G' is the elastic modulus, G'' the viscous modulus and \mathbf{g}_0 the initial strain.

Small amplitude oscillatory parameters are regularly performed in commercial rheometer. Results are repetitive and reliable, but there is a major limitation: high shear rates are normally not possible to be reached. Master curve reduction techniques are alternative ways to expand the shear rate range.

c) Shear free flows

$$\tau_{11} - \tau_{33} = \mathbf{m}_{e1} \dot{\mathbf{e}} \quad (7)$$

$$\tau_{22} - \tau_{33} = \mathbf{m}_{e2} \dot{\mathbf{e}} \quad (8)$$

where τ_{11} , τ_{22} and τ_{33} are normal stress components, \mathbf{m}_{e1} and \mathbf{m}_{e2} are the first and second extensional viscosities and $\dot{\mathbf{e}}$ is the strain rate.

Extensional properties measurement is, a major research area in rheology. Different techniques have been proposed, each of them providing different results. Two commercial equipments using different techniques are available (Fuller¹⁹ and Spiegelberg and McKinley²⁰).

EXPERIMENTAL WORK

The main goal of this study is to identify and equate rheological phenomena governing fluid invasion. The strategy to develop this concept was to flow different kinds of polymeric solutions through consolidated inert porous media (ceramic disks of 6.35 cm diameter and 0.635 cm thickness). The experiment was run in static conditions and under differential pressures up to 300 psi, typical of over balanced drilling operations. The porous medium is supported by a high permeability screen in order to minimize

friction losses at the fluid discharge system. The fluid volume which flowed through the porous medium is monitored along the time and its rheological properties evaluated at test temperature.

In order to isolate rheological effects from bridging effects, the strategy was to work with solids free polymeric solutions. In this case, different flow behaviors observed in experimenting different polymeric solutions would be attributed only to rheological effects. Since no solids are present, no relevant external filter cake will be formed, resulting that a static filtration approach would be enough for rheological effects identification.

PHPA and XC solutions were flowed through porous media at constant pressure differential, establishing the volume vs. time curve. This process was repeated in a sequence of increasing and decreasing pressure differentials (40 up to 200 psi and then down to 40 psi again) and of decreasing and increasing pressure differentials (200 psi down to 40 psi and then up to 200 psi again). The idea was to check the repeatability of experimental flow rates measured for the same pressure differential reached in different manners. After that, glycerin was flowed through each of the porous media at the same pressure increase/decrease sequence in order to check the porous media permeability after the polymer flow. Finally, some extra tests were run with an additional step: before the polymeric solution, glycerin was flowed through the porous medium to check its original permeability. After that, the same procedure described was adopted. The following set of tests was performed:

1. 16 lb/bbl PHPA solution in water and glycerin/water mixture in the low permeability disk.
2. 4 lb/bbl XC solution in water and glycerin/water mixture in the low permeability disk.
3. 4 lb/bbl XC solution in water and glycerin/water mixture in the high permeability disk.

4. Glycerin/ water mixture, 16 lb/bbl PHPA solution and glycerin/water mixture in water in the low permeability disk.
5. Glycerin/ water mixture, 3 lb/bbl XC solution and glycerin/water mixture in water in the low permeability disk.

From the collected volume vs. time curve, for a given experiment, the relevant parameters were calculated. The samples were characterized in commercial rheometers under simple shear and low amplitude oscillatory experiments. Besides, extensional properties were estimated at an opposed jet rheometer (Rheometrics RFX). All the rheological experiments covered were performed at the same temperature and shear rate range than the equivalent filtration tests.

Fig. 1 illustrates some of the results for the PHPA polymer solution flowing in the lower permeability disk, as functions of the imposed differential pressure during the increase/decrease sequence. The good repeatability in the several results for each pressure differential suggest that non-Darcy effects should be considered to explain the deviations discussed in the previous item.

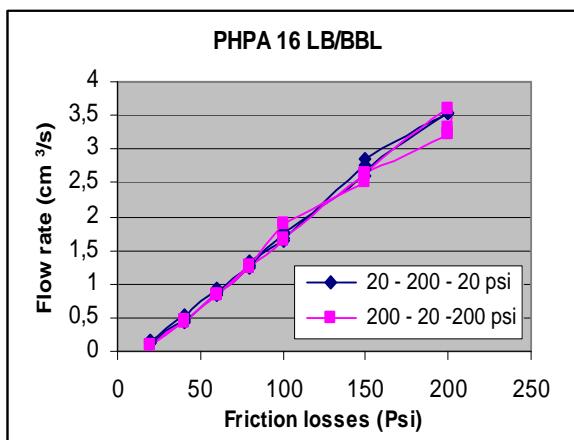


Figure 1 - Repeatability of PHPA filtration test

Average results of permeability recovery after polymer injection, highlighted in Table 1, show that there is always a portion of polymer that does not leave the porous media due to adsorption effects (Chiappa²¹).

Table 1 – Disk permeability to glycerin before and after polymer injection

Test	K _{Before} (mD)	K _{After} (mD)
1	538	408
2	538	457
3	3820	2430
4	570	548
5	565	361

Some considerations are now made concerning viscoelastic measurements. Experimental results for the solutions are also presented.

N_1 measurements: Fig. 2 shows the experimental N_1 / t ratio in function of shear rates for the PHPA and XC solutions used in the filtration tests. There is a discussion in the literature concerning the magnitude of N_1 when compared to t . The presented results are in accordance with Dauben and Menzie¹⁶ who affirm that the normal stresses of viscoelastic fluids are often as large as or larger than the tangential stresses associated with viscosity.

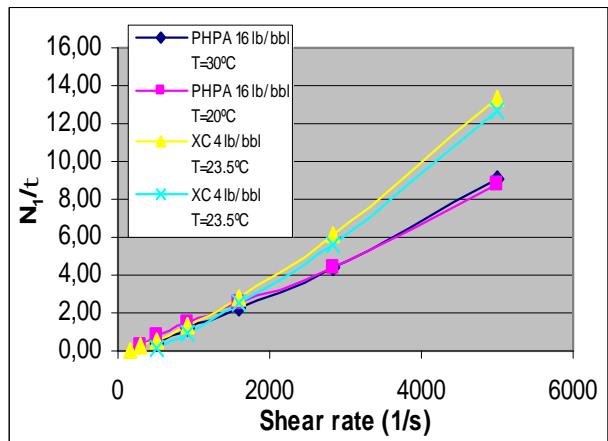


Figure 2 - N_1 evaluation

Oscillatory measurements: Fig. 3 shows the linear viscoelastic parameters obtained in the low amplitude oscillatory tests for the 3 different polymeric solutions. Even with the variable reduction approach, maximum shear stresses reached 170 s^{-1} . This fact difficult the modelling of most filtration experiments which occurred at higher shear rates.

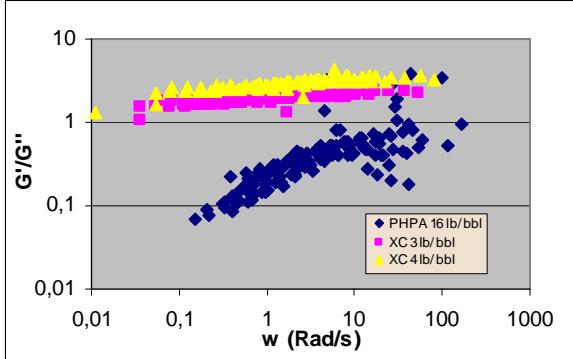


Figure 3 - Oscillatory measurements

Extensional viscosity measurements: Fig. 4 shows the Trouton ratio for the 3 solutions, obtained by the opposed jet nozzles rheometer. Further effort is required to obtain results with different technique rheometers in order to gain confidence in the values. An interesting results, also observed by Chauveteau et al.²², is that PHPA solutions observed both shear thinning and extensional thickening behaviors which can explain higher deviations at high pressure differential tests. XC solutions presented both shear and extensional thinning behavior.

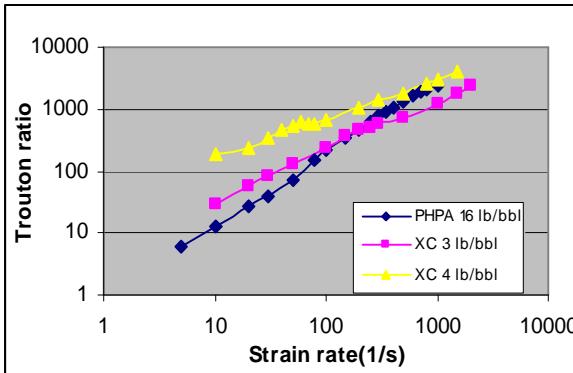


Figure 4 – Trouton ratio vs Strain rate

DISCUSSION

Experimental results indicate that, especially for the highly concentrated PHPA and XC solutions, considerable deviation in relation to Darcy Law predictions are observed. There is a clear tendency that such deviations are higher at the higher pressure differential tests and, consequently, where higher shear rates are experimented. For the CMC solutions and for the low concentration

PHPA and XC solutions, the deviations are much smaller and can be attributed to experimental uncertainties.

In order to explain additional resistive forces occurring when a viscoelastic fluid flows through a porous media, a general proposition of Massarani and Silva Telles¹⁶ will be considered. Such expression accounts for the high velocity (Forchheimer) and normal stress differences effects, according to the following expression:

$$\left(\frac{\Delta P}{L} \right) = \frac{m_{ef}(I^*)}{K} \left(1 + \frac{C \cdot \sqrt{K} \cdot r \cdot q}{m_{ef}(I^*)} + \frac{C_1 \cdot N_1(I^*)}{t(I^*)} + \frac{C_2 \cdot N_2(I^*)}{t(I^*)} \right) \cdot q \quad (9)$$

where K, C, C₁ and C₂ are the adjusted coefficients. K and C are related to the porous structure of the medium while C₁ and C₂ may be related to both the fluid and the porous medium. Since no measurements of N₂ are available, the last term will be neglected in the following analysis. Similar expressions can be proposed to correlate the additional resistive forces as functions of the oscillatory parameters and the extensional parameters, as follows:

$$\left(\frac{\Delta P}{L} \right) = \frac{m_{ef}(I^*)}{K} \cdot \left(1 + \frac{C_3 G'(I^*)}{G''(I^*)} \right) \cdot q \quad (10)$$

$$\left(\frac{\Delta P}{L} \right) = \frac{m_{ef}(I^*)}{K} \cdot \left(1 + \frac{C_4 m_e(\epsilon)}{m_{ef}(I^*)} \right) \cdot q \quad (11)$$

C₃ and C₄ are the new adjusted coefficients. A Power Law approach was considered to fit the shear stress, the N₁, the G' and G'' and the extensional stress curves, according to the following expressions:

$$t = M I^n \quad (12)$$

$$N_1 = M_1 I^{n_1} \quad (13)$$

$$G' = M_2 I^{n_2} \quad (14)$$

$$G'' = M_3 I^{n_3} \quad (15)$$

$$t_E = M_4 \epsilon^{n_4} \quad (16)$$

Where (M and n), (M_1 and n_1), (M_2 and n_2), (M_3 and n_3), (M_4 and n_4), are the power law coefficients for the several material functions. Table 2 shows the coefficients for each of the fluids tested in Phase 2, as well as the adequacy of power law to the experimental rheological data (obtained at 23 °C). R is the correlation coefficients. Except from the oscillatory experiments, all the others fitted properly to the power law approach.

Table 2 - Adequacy of Power Law approach to the experimental rheological data obtained at 23 °C

XC 3.0 LB/BBL		XC 4.0 LB/BBL		PHPA 16 LB/BBL	
M (Pa.s ⁿ)	0,739	M (Pa.s ⁿ)	3,818	M (Pa.s ⁿ)	0,924
n	0,442	n	0,196	n	0,539
R	0,992	R	0,995	R	0,991
M_1 (Pa.s ⁿ)	7,0E-06	M_1 (Pa.s ⁿ)	1,0E-06	M_1 (Pa.s ⁿ)	7,3E-03
n_1	2,143	n_1	2,341	n_1	1,329
R	0,998	R	0,996	R	0,998
M_2 (Pa.s ⁿ)	7,604	M_2 (Pa.s ⁿ)	6,283	M_2 (Pa.s ⁿ)	0,011
n_2	0,337	n_2	0,335	n_2	0,947
R	0,964	R	0,961	R	0,893
M_3 (Pa.s ⁿ)	4,185	M_3 (Pa.s ⁿ)	3,354	M_3 (Pa.s ⁿ)	0,058
n_3	0,230	n_3	0,229	n_3	0,569
R	0,930	R	0,933	R	0,872
M_4 (Pa.s ⁿ)	5,301	M_4 (Pa.s ⁿ)	43,4	M_4 (Pa.s ⁿ)	0,824
n_4	0,809	n_4	0,616	n_4	1,186
R	0,992	R	0,991	R	0,997

Eq. 9 and 10 can be coupled with Eq. 2 and reduced to the Darcy's Law, The Forchheimer Law and for the several viscoelastic approaches. Considering a power law fluid, which material functions are represented by (M_i , n_i) and (M_j , n_j), the following expressions can be derived for each case.

- Darcy :

$$\left(\frac{\Delta P}{L} \right) = \frac{M \cdot q^n}{K^{\frac{n+1}{2}}} \quad (17)$$

- Forchheimer :

$$\left(\frac{\Delta P}{L} \right) = \frac{M \cdot q^n}{K^{\frac{n+1}{2}}} + \frac{C \cdot r \cdot q^2}{\sqrt{K}} \quad (18)$$

- Generalized form for the Viscoelastic approaches :

$$\left(\frac{\Delta P}{L} \right) = \frac{M \cdot q^n}{K^{\frac{n+1}{2}}} \cdot \left[1 + C_i \cdot \frac{M_i}{M_j} \cdot \left(\frac{q}{\sqrt{K}} \right)^{n_i - n_j} \right] \quad (19)$$

Table 3 shows the estimated values of K , C and C_i for the 3 proposals represented by Eqs. 17, 18 and 19, including the several options for the viscoelastic effects. Besides, the relevant statistical parameters are also presented. These results reflect the fluids tested alone and the combination of tests in the same plug (fluid and glycerin).

The adequacy of the Forchheimer law would be conditioned to the fact that K and C would be only functions of the porous medium. Since the values of K and C are different for the polymer solutions and for the glycerin, which have flowed, in the same plug, the hypothesis seems to be inadequate. Besides, when the data of a polymeric solution and of the glycerin in the same plug are treated together the correlation coefficient decreases to unacceptable values. Forchheimer Law is normally applied in flows where Reynolds Number reaches 0.1, which is much larger than the values calculated for these experiments.

The adequacy of the viscoelastic approaches is also shown in Table 3. Correlation coefficients seemed to be improved a lot if compared to Darcy Law (in all tests), for the first normal stress and for the extensional viscosity approaches. Besides, this approach does not present any restrictions in relation to the coefficients, since C_1 (unlike Forcheimer's Law C) may be both fluid and porous medium dependant. In some cases, however, the standard deviation in the coefficients approaches the same order of

Table 3 - Parameters estimation results

Fluid	Darcy Law		Forchheimer Law			$\frac{N_1}{t}$		$\frac{m_e}{m}$		$\frac{G'}{G''}$				
	K (mD)	R	K (mD)	C	R	K (mD)	C ₁	R	K (mD)	C ₃	R	K (mD)	C ₄	R
PHPA	168	0.94	267	2.25E+04	0.99	389	0.37	0.99	499	1.45E-02	0.99	949	1.15E+00	0.97
PHPA + Glycerin	300	0.63	468	4.64E+04	0.93	407	0.40	0.98	484	1.40E-02	0.98	194	-9.30E-03	0.87
XC	53	0.48	70	3.44E+02	0.55	67	7.70E-04	0.55	103	1.10E-03	0.52	186	2.47E-01	0.49
XC + Glycerin	392	0.33	440	2.39E+03	0.49	423	0.047	0.39	543	8.80E-03	0.69	456	6.10E-01	0.70
XC	419	0.65	901	3.91E+02	0.97	764	0.012	0.97	2056	5.80E-03	0.92	3.75E+07	3.14E+02	0.79
XC + Glycerin	2321	0.66	2657	1.01E+03	0.97	2340	1.28E-02	0.95	2927	8.60E-03	0.98	1892	-1.00E-05	0.67
PHPA	339	0.93	511	1.95E+04	0.99	734	0.56	0.99	901	2.04E-02	0.99	248	9.20E-03	0.95
PHPA + Glycerin	356	0.94	400	1.02E+04	0.96	376	6.60E-2	0.95	390	2.20E-03	0.95	224	-7.50E-03	0.82
XC	196	0.82	355	9.53E+02	0.94	328	1.30E-2	0.92	2128	3.25E-02	0.99	6.98E+05	1.08E+02	0.98
XC + Glycerin	534	0.72	578	1.70E+03	0.92	562	4.40E-2	0.92	658	8.20E-03	0.94	356	-4.70E-03	0.49

magnitude of the coefficient itself. The linear viscoelastic parameters, although easily and reliably obtained, proved to be inadequate for the analysis: although correlation coefficients are reasonable, the deviation on each parameter is extremely high resulting in meaningless coefficients.

FINAL REMARKS

Experimental results show that Darcy law was capable to reproduce the flow of low concentration PHPA and XC solutions through porous media. For higher concentrations of XC and PHPA deviations from Darcy law becomes larger, especially at the high differential pressure tests.

This fact leads to the conclusion that viscoelastic effects play a relevant role in such conditions. A first attempt to model the flow as function of the ratio between first normal

stress difference and shear stress (N_1/t) or as a function of the Trouton ratio resulted in higher correlation coefficients besides reliable adjusted coefficients. However, more confidence on N_1 and extensional viscosity measurements is still required.

Further steps on modeling include the consideration of the local extensional viscosity effects on friction losses, as well as the two phase flow (oil – polymeric solution) inside the porous medium.

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