Fluid Displacement in a Tube

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ABSTRACT

The displacement of Newtonian fluids by non-Newtonian fluids in a horizontal cylindrical tube has been investigated both theoretically and experimentally. Theoretical expressions for the breakthrough time, a function of the constitutive constants of the fluids and of the flow parameters and geometry, are derived when the displacing fluids are either of the inelastic viscous or nonlinear viscoelastic type. The dynamics of the displacement process is studied experimentally using the photometry method and an oil field spacer fluid and glycerol/water mixtures as the displacing and displaced fluids, respectively.

INTRODUCTION

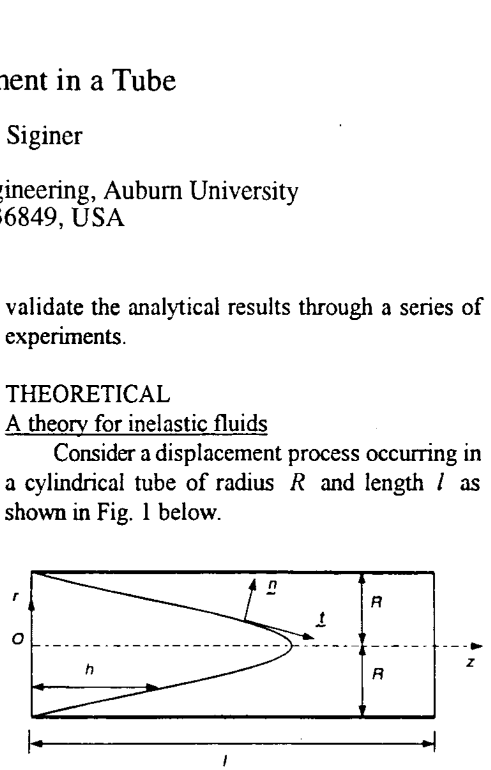
The displacement of one fluid by another is a process of common occurrence in oil fields. The displacement processes are complex, involving the time-dependent motion of multiple Newtonian and non-Newtonian fluids with different densities and rheologies, flowing in channels of different configurations such as cylindrical tubes, concentric and eccentric annuli, variable cross section channels, etc.

The most commonly used parameters for defining the ability of a given fluid to displace another are the breakthrough time and the displacement efficiency. Breakthrough time is defined as the time it takes for the displacing fluid to appear at the outlet of the tube. The differences in densities and viscosities of the fluids involved, and flow rates play a predominant role in the displacement process and in determining the breakthrough time. This paper discusses the displacement process in a horizontal cylindrical tube from both a theoretical and experimental point of view. We present theoretical results for the breakthrough time when an inelastic or a viscoelastic fluid displaces a Newtonian liquid, and validate the analytical results through a series of experiments.

THEORETICAL

A theory for inelastic fluids

Consider a displacement process occurring in a cylindrical tube of radius R and length l as shown in Fig. 1 below.



We assume that flow is laminar and steady on either side of the interface, and that the interface is stable and smooth. The particular shape of the interface is not very important to this investigation. Consequently, we neglect surface tension effects. The displacement process occurs under stable conditions. The pressure gradient is constant, and end effects are negligible. The displaced fluid is Newtonian with viscosity μ. The displacing fluid is inelastic and of the linear fluidity type with fluidity φ,

$$[\mu(J_2)]^{-1} = \varphi(I_2) = \varphi_0 + \theta(0.5\,|I_2|)^{0.5} ,$$

where J_2 and I_2 represent the second invariants of the rate of deformation and stress tensors, respectively.

In the case of the displacement process in a horizontal round pipe, we use an approximate approach which assumes that the velocity profiles of both liquids are fully developed. For long tubes, that is, for large l/R ratios, which is the case in our experiments, this assumption gives excellent results. We write,

$$\underset{\sim}{u}_i = V_i(r)\underset{\sim}{e}_z \ , \quad V_{i,r} = -\varphi_i(\tau)\tau \ , \quad \tau = -\frac{r}{2}P_{,z} \ ,$$

$$\tau_w = -\frac{R}{2}P_{,z} \ , \quad i = 1,2 \ ,$$

where τ_w denotes the wall shear stress to obtain the velocity distribution for displacing and displaced fluids as,

$$V_1 = \frac{\varphi_o(P_1 - p)R^2(1-\xi^2)}{4z} + \frac{\theta(P_1 - p)^2R^3(1-\xi^3)}{12z^2} \ ,$$

$$V_2 = \frac{\varphi_2(p - P_2)R^2(1-\xi^2)}{4(l-z)} \ , \quad \xi = \frac{r}{R} \ ,$$

where P_1 and P_2 are the pressures at the left and right ends of the horizontal tube ($P_1 > P_2$), respectively, and p stands for the continuous one dimensional pressure field.

The kinematic and dynamic conditions at the interface require that,

$$V_1(r)|_{z=h} = V_2(r)|_{z=h} \ ,$$

$$\overset{1}{S}_{z\theta} - h_{,r}\overset{1}{S}_{r\theta} = \overset{2}{S}_{z\theta} - h_{,r}\overset{1}{S}_{r\theta} \ ,$$

$$h_{,r}(\overset{1}{S}_{zz} - \overset{1}{S}_{rr}) + (1 - h_{,r}^2)\overset{1}{S}_{rz} = h_{,r}(\overset{2}{S}_{zz} - \overset{2}{S}_{rr}) + (1 - h_{,r}^2)\overset{2}{S}_{rz} \ ,$$

where h represents the moving interface, and upper and lower indices 1 and 2 refer to displacing and displaced fluids, respectively. We determine the pressure P along the interface,

$$P = p_1|_{z=h} = p_2|_{z=h} = \frac{P_1\left(1 + \frac{\psi\lambda}{y}\right) + \frac{P_2\zeta y}{(1-y)}}{1 + \frac{\psi\lambda}{y} + \frac{\zeta y}{(1-y)}} \ ,$$

$$y = \frac{h}{l} \ , \quad \zeta = \frac{\varphi_2}{\varphi_o} \ , \quad \psi = \frac{\theta R(P_1 - P_2)}{3\varphi_o l} \ , \quad \lambda = \frac{(1-\xi^3)}{(1-\xi^2)}$$

Using this result in V_1 and integrating yields the breakthrough time on the axis,

$$T = \frac{0.5 + (0.5 - \psi)\zeta}{\psi\zeta} + \psi \ln\frac{\psi + 1}{\psi} \ , \quad T = \frac{3\varphi_0^2 R t_l}{4L\theta} \ ,$$

with t_l denoting the dimensional time.

A theory for viscoelastic fluids

We assume that the displacing fluid is of the Oldroyd-B type,

$$\underset{\sim}{S} + \lambda_1\overset{\nabla}{\underset{\sim}{S}} = -\mu_0(\underset{\sim}{d} + \lambda_2\overset{\nabla}{\underset{\sim}{d}}) \ , \ \overset{\nabla}{(\cdot)} = \frac{D}{Dt}(\cdot) - [\underset{\sim}{d}(\cdot) + (\cdot)\underset{\sim}{d}]$$

where μ_0, $\underset{\sim}{d}$, $\underset{\sim}{S}$, λ_1 and λ_2 are the zero shear viscosity, the rate of deformation and the extra-stress tensors and the relaxation and retardation times, respectively, with $0 \le \lambda_2 < \lambda_1$. The velocity field in a straight, round tube is given by

$$V_1 = -\frac{\lambda_2}{2\mu_0}\frac{P_1 - p}{Z}A(r) \ , \quad A(r) = \lambda_2\left(e^{r/\lambda_2} - e^{R/\lambda_2}\right) + \frac{R^2 - r^2}{2\lambda_2} + R - r \ .$$

We compute the pressure P at the interface as,

$$P = \frac{P_1 B(h)A(r) - C(r,h)P_2}{B(h)A(r) - C(r,h)} \ , \quad B(h) = \frac{\lambda_2}{2\mu_0 h} \ ,$$

$$C(r,h) = \frac{R^2 - r^2}{4\mu_2(l-h)} \ ,$$

and the breakthrough time t_l as a function of r,

$$t_l = \frac{(a-1)l^2}{2b} \ ,$$

$$a = \frac{2\mu_2\lambda_2 A(r)}{\mu_0(R^2 - r^2)} \ , \quad b = \frac{\lambda_2(P_1 - P_2)A(r)}{2\mu_0} \ .$$

EXPERIMENTAL AND DISCUSSION

Experiments were run in a closed circulation loop. The test channel is a cylindrical glass tube 2050 mm long and 10 mm in inside diameter. The dynamics of the displacement process in the channel is studied by the photometry method. Newtonian liquids to be displaced are water and glycerol/water mixtures. The displacing fluid is a polyacrylamide based spacer fluid. The densities of the displacing and displaced fluids are almost equal in our experiments as they may be in most field operations. Experiments were run at 22.6°C.

Experiments were run with three values of the viscosity ratio $\zeta = 1,2,5$ with varying ψ and four values of $\psi = 1,2,4,6$ with varying ζ. Fig. 2 shows the variation of the dimensionless breakthrough time T with the parameter ψ, a function of the pressure gradient, tube size and the constitutive constants of the displacing fluid, for fixed zero shear viscosity ratio ζ of the displacing and displaced fluids. The predictions using the inelastic theory are also shown in Fig. 2. Viscosity ratio ζ plays a predominant role in determining the breakthrough time. That is particularly noticeable in the region $\zeta<4$. For instance, breakthrough time is approximately cut in half when viscosity ratio ζ increases from 0.5 to 1 at any value of ψ. Further, the breakthrough time increases dramatically with very small decreases in the viscosity ratio ζ when $\zeta<0.5$ and $\psi>4$. Variations in the viscosity ratio ζ in the region $\zeta>4$ do not affect the breakthrough time significantly. Breakthrough time T also shows strong dependence on the parameter ψ which has an increasingly larger effect on T with decreasing values of ψ at any value of ζ, in particular when $\psi<6$. The breakthrough time shows very large increases for a small decrease in ψ at any fixed ζ when $\psi<2$. In particular, the increases in T are dramatic for a quite small change in ψ when $\psi<1$ and $\zeta>3$. T does not change noticeably with increasing ψ in the region $\psi>6$. Experimental data is better predicted by the viscoinelastic model at larger values of the viscosity ratio ζ.

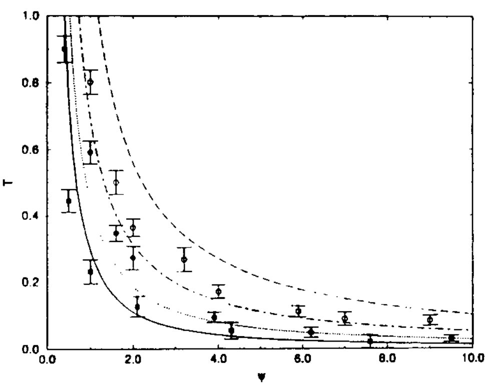


Fig. 2 Comparison of the experimental data and theory for the dimensionless breakthrough time T as a function of the parameter ψ. It is assumed that the displacing fluid is described by an inelastic viscous structure. Full curves correspond to the predictions of the inelastic theory: (--) $\zeta=0.5$, (-·-) $\zeta=1$, (···) $\zeta=2$ and (—) $\zeta=5$. Symbols denote experimental data: (○) $\zeta=1$, (●) $\zeta=2$, (■) $\zeta=5$.

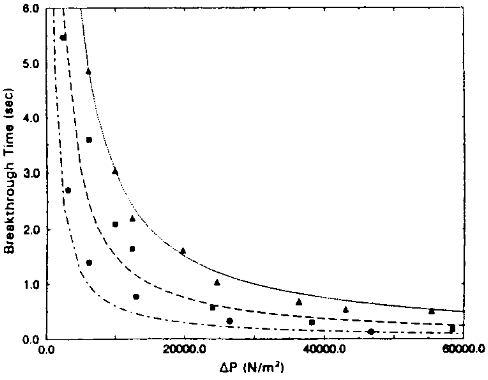


Fig. 3 Comparison of the experimental data and theory for the dimensional breakthrough time t_l on the tube axis as a function of the pressure difference ΔP. It is assumed that the displacing fluid is described by a viscoelastic structure of the Oldroyd-B type : (····) $\zeta=1$, (---) $\zeta=2$, (-·-) $\zeta=5$. Symbols denote experimental data: (▲) $\zeta=1$, (■) $\zeta=2$, (●) $\zeta=5$.

Theoretical predictions for the dimensionless breakthrough time t_l on the tube axis, when the displacing fluid is assumed to be viscoelastic of the Oldroyd type, are shown in Fig. 3 together with experimental results. The predictions are better for small values of the viscosity ratio at all values of the pressure difference. For large values of the viscosity ratio, viscoelastic predictions undershoot the displacement time by as much as 20% when $\Delta P\sim 10^4$ Pa. For larger pressure gradients, $\Delta P\sim 5\times10^4$ Pa, breakthrough time is reasonably well predicted at all values of the viscosity ratio. One is drawn to the inescapable conclusion that a viscoelastic description of the constitutive behavior of the displacing fluid yields better predictions at relatively small values of the viscosity ratio around $\zeta=1$, whereas a viscoinelastic description succeeds at matching the experimental results better at higher values of the viscosity ratio. We remark that in oil field displacement operations, relatively high viscosity ratios may be desirable for the efficiency of the operation as high ζ implies a relatively flat interface and a more uniform displacement process.