Flow in Non-Homogeneous Porous Media and Novel Effects

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ABSTRACT

Experimental results together with novel effects concerning the flow of non-Newtonian liquids in two porous media of different permeabilities and same porosity arranged in series are reported for the first time in this paper. We determine that energy requirements may be different by as much as 25-35% depending on the Reynolds number when the flow direction is reversed. We also develop two theories for inelastic and viscoelastic fluids, respectively, to predict the deserved effects.

INTRODUCTION

The dynamics of the flow through porous media is increasingly of pivotal importance to many petroleum engineering applications such as acidizing, fracturing, secondary recovery methods (water and gas flooding, steam injection, in situ combustion), gas cycling, etc. Polymer solutions of different concentrations and rheological properties are increasingly and widely used in these applications.

Most investigations in the literature are concerned with flow through homogeneous porous media. However, in oil engineering applications, flow through heterogeneous porous media is encountered frequently. In this paper, we report experimental results concerning the flow of Newtonian, viscoelastic and viscoinelastic liquids in a porous medium with a step change in permeability, that is, two porous media of different permeabilities and same porosity in series. The results obtained can be heuristically extended to N porous media in series and consequently to an anisotropic, nonhomogeneous medium. We show that the energy loss is higher if the polymeric solution flows first through the porous medium with the smaller permeability rather than through the section of the cylinder with the larger permeability. The difference in energy requirements increases with increasing Reynolds number and may be as high as 25-35% for Reynolds numbers of $O(1)$. This is a novel effect not observed for Newtonian and highly shear thinning inelastic fluids flowing through the same configuration. Energy requirements for the same volume flow rate are much higher than a Newtonian fluid of the same zero shear viscosity as the polymeric solution. Energy loss increases with increasing Reynolds number at a fixed concentration to level off at a Reynolds number of $O(1)$. At a fixed Reynolds number, the loss is a strong function of the concentration and shows large increases with increasing concentration. For shear-thinning oil field spacer fluids $De \sim 0.1$ represents a good criterion for the onset of elasticity effects. For solutions of polyacrylamide $De \sim 0.1$ corresponds approximately to the flow rate at which pressure drop starts becoming dependent on the flow direction.

Expressions for the friction factor and the resistance coefficient as a function of the Reynolds number have been developed for inelastic and viscoelastic fluids based on the KPK (Kutateladze-Popov-Kapakhpasheva) and eight constant Oldroyd models, respectively. The behavior of the former and the latter, as represented by oil field spacer fluids and aqueous solutions of polyacrylamide, is predicted qualitatively except the difference in energy requirements when the flow direction is reversed in the case of the latter.

THEORETICAL ANALYSIS

A theory for inelastic fluids

To describe the shear rate dependent viscosity, we use the concept of fluidity $\varphi(\tau)$ defined as the reciprocal of the viscosity, and conceived of as depending on the shear stress τ, Kutateladze et al [1].

$$\varphi(\tau) = \varphi_0 + \theta(\tau - \tau_1) - \frac{\theta^2}{2}\frac{(\tau - \tau_1)^2}{\varphi_\infty - \varphi_0} + O(\theta^3) .$$

$$\varphi_0 \le \varphi \le \varphi_\infty \; ; \; \varphi \to \varphi_\infty , \tau \to \infty \; ; \; \varphi = \varphi_0 , \tau \le \tau_1 .$$

θ is the structural fluidity coefficient. Keeping only the first two terms in this expansion yields the linear fluidity relationship. Kutateladze et al [1] show that the linear fluidity law,

$$\varphi = \varphi_0 + \theta\tau \; , \; \tau_1 \sim 0 , \qquad (1)$$

represents the function $\varphi(\tau)$ rather well for diverse fluids in the range of shear stresses of practical interest. Using (1), we determine that the pressure drop represented in terms of the resistance coefficient Λ, a function of the friction factor f and the Reynolds number Re, is given by

$$\Lambda = Re \cdot f , \; f = \frac{\Delta P D_p \epsilon^3}{\rho u_0^2 L (1 - \epsilon)} , \; Re = \frac{D_p u_0 \rho \varphi}{1 - \epsilon} ,$$

$$Re = \beta (1 + 0.13\psi)(1 + 0.08\psi) ,$$

$$f = \frac{150}{\beta (1 + 0.13\psi)^2} , \; \Lambda = \frac{150 (1 + 0.08\psi)}{1 + 0.13\psi} ,$$

$$\beta = \frac{\varphi_0^2 \epsilon^3 D_p^3 \rho \Delta P}{150 (1 - \epsilon)^3 L} , \; \psi = \frac{\theta D_p \epsilon \Delta P}{\varphi_0 (1 - \epsilon) L} ,$$

where D_p, ϵ, u_0 and L represent the average particle diameter, the porosity, the superficial velocity and the test section length, respectively.

A theory for viscoelastic fluids

An 8-constant Oldroyd model is used. This model predicts,

$$\frac{\eta}{\eta_0} = \frac{1 + \sigma_2 \dot{\gamma}^2}{1 + \sigma_1 \dot{\gamma}^2} ,$$

for the viscosity η at a given shear rate $\dot{\gamma}$. η_0, σ_1 and σ_2 are the zero shear rate viscosity and material parameters, respectively. We determine that the pressure drop ΔP over the length L is given by,

$$\Delta P = \frac{6\eta_0 L A}{R_h}\sqrt{\frac{X}{\sigma_1}} , \; R_h = \frac{D_p \epsilon}{6(1 - \epsilon)} ,$$

$$X = \sigma_1 \dot{\gamma}^2 |_{r=R} , \; n = \frac{\sigma_2}{\sigma_1} , \; A = \frac{1 + nX}{1 + X} ,$$

where R_h is the hydraulic radius. The resistance coefficient Λ, the friction factor f and the Reynolds number Re are related through,

$$f = \frac{486\,\eta_0 C^2 B^2 A}{\rho R_h^2}\sqrt{\frac{\sigma_1}{X}} , \; \Lambda = \frac{108 C X^2}{2X^2 - A^{-3} F}$$

$$Re = \frac{\rho R_h^2 B}{9 C A \eta_0}\sqrt{\frac{X}{\sigma_1}} , \; B = 1 - \frac{F}{2X^2 A^3} ,$$

$$F = \frac{1}{2} n^3 X^3 - 3n^2(n-1)X + 3n(n-1)(2n-1)$$
$$\ln(1 + X) - \frac{1}{2} X \left(\frac{n-1}{1+X}\right)^2 [6n + (7n-1)X] ,$$

where C is the tortuosity factor. We find that for viscoelastic fluids C is a function of the elasticity of the fluid and assumes increasingly larger values with increasing elasticity.

EXPERIMENTAL

Experiments were run in a porous medium represented by a stainless steel flow cell with an internal diameter of 4.5 cm and a length of 30 cm, packed with glass spheres of 1000±50 and 3000±50 microns in series. Spheres of different diameters fill each half of the tube. Newtonian liquids used in the experiments are water and a glycerol-water solution with a viscosity of 0.0184 Pa·s at 20°C. The non-Newtonian liquids investigated are a polyacrylamide based oil field spacer fluid and aqueous polyacrylamide solutions of 1% and 2% concentration by weight prepared with distilled water and polyacrylamide of molecular weight 5 × 10^6. The oil field spacer fluid contains 0.6% polyacrylamide by weight with several different proprietary additives.

RESULTS AND DISCUSSION

Two sets of experiments have been conducted for each fluid: Tests (I): Flow proceeds through the porous medium with the smaller permeability towards the porous medium with the larger permeability. Tests (II): Flow proceeds through the porous medium with the larger permeability towards the porous medium with the smaller permeability.

Experimental results together with theoretical predictions are shown in Fig. 1.

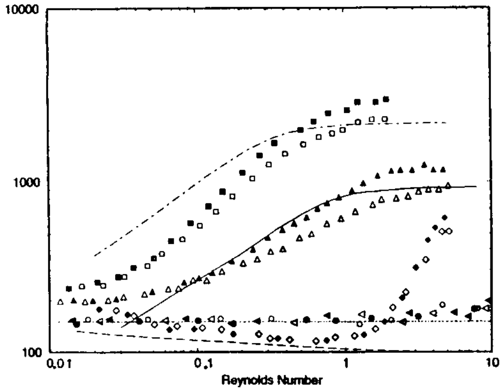

Fig. 1 Resistance coefficient as a function of the Reynolds number. Tests I: (●) - distilled water, (◂) - glycerol/water, (▲) - 1% PAA, (■) - 2% PAA, (♦) - spacer fluid; Tests II: (○) - distilled water, (◃) - glycerol-water; (△) - 1% PAA, (□) - 2% PAA, (◇) - spacer fluid. Solid curves correspond to theoretical predictions: (—) 1% PAA, $\eta_0 = 0.133\,Ns/m^2, n = 0.286, \sigma_1 = 3\cdot10^{-5} s^{-2}$, $C = 1.3$; (- - -) 2% PAA, $\eta_0 = 0.150\,Ns/m^2$, $n = 0.250, \sigma_1 = 3\cdot10^{-5} s^{-2}, C = 3.0$; (– – –) spacer fluid, $\theta/\varphi_0 = 0.06\,m^2/N$; (·····) Newtonian fluids, distilled water and glycerol/water solution.

We find that the energy loss is considerably higher if a polymeric solution flows first through the porous medium with the smaller permeability rather than through the section of the cyliner with the larger permeability. This is a novel effect not observed for Newtonian and highly shear thinning inelastic fluids flowing through the same configuration. The difference in energy requirements for the same volume flow rate increases with increasing Reynolds numbers at any concentration when the flow direction is switched from $K_1 \rightarrow K_2$ to $K_2 \rightarrow K_1$ $(K_1 > K_2)$, where K_i represents the permeability. It reaches an almost constant value at $Re \sim O(1)$. At $Re \sim O(1)$, it is as much as 25% and 35% for 1% and 2% solutions of polyacrylamide, respectively. At all Reynolds numbers, the pressure drop required for the same volume flow rate is much higher than the Newtonian liquid of the same zero shear viscosity. Energy loss increases with increasing Reynolds numbers to level off at a Reynolds number of $O(1)$. The pressure drop required at that Reynolds number is an order of magnitude larger than the pressure drop for the Newtonian liquid, and increases with increasing concentration. At a fixed Reynolds number, the loss is a strong function of the concentration and shows large increases with increasing concentration.

We find that for two Newtonian liquids, water and a water/glycerol solution, the resistance coefficient is constant when inertial effects are negligible, $Re < 10$. A highly shear-thinning oil-field spacer fluid requires less energy (smaller resistance coefficient) than a Newtonian fluid for the same volume flow rate for Reynolds numbers between 0.08 and 1.5. The friction factor and the resistance coefficient for Newtonian and inelastic (shear-thinning) fluids do not depend on the flow direction as we obtain the same data when the flow direction is reversed. Elastic effects start becoming important at a critical Deborah number of 0.1. The pressure drop required for the same volume flow rate is higher than the Newtonian case for $Re > 2$, and increases rapidly with increasing Reynolds numbers.

The Newtonian behavior is well predicted theoretically when inertial effects are negligible. Two theories built on the inelastic KPK (Kutateladze-Popov-Kapakhpasheva) and viscoelastic Oldroyd models show only qualitative agreement with experimental data for the nonlinear fluids used. But, predicting the difference in energy requirements for viscoelastic fluids even qualitatively when the flow direction is reversed remains a challenge. Further details can be found in Siginer & Bakhtiyarov [2].

REFERENCES

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