Laminar Non-Newtonian Flow in an Eccentric Annulus, a numerical solution.

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ABSTRACT

A numerical program is developed to study the flow of a generalised Newtonian fluid in an eccentric annulus. The result of simulations is presented. The program is verified with experimental data for both concentric and eccentric annuli's.

INTRODUCTION

Motivation for this work is the need to know the downhole conditions accurately during oil or gas well drilling. In an eccentric annulus the velocity profile is a key factor in the determination of pressure loss, wall tension and related quantities.

The primary function of the drilling mud is to control the pressure in the well. It is typically a mixture with a high content of polymer and particles. The drilling mud is modelled as a generalised Newtonian fluid.

It normal drilling operations the drilling mud circulates from the rig floor down to the bottom of the hole inside the drill string. From the bottom hole up to the rig floor again the drilling mud flows in the annulus between the drill string and the well bore walls. Eccentric placement of the drill string may change the character of the flow and it may render the down hole conditions unpredictable.

Several authors have presented numerical works on the flow of generalised Newtonian fluids in an eccentric annulus. Guckes[1] presents results for both power law and Bingham fluids. Haciislamoglu and Langlinalis[2] presents velocity profiles for several non-Newtonian fluids and in addition presents a correlation based on the numerical results for the pressure drop of a power law fluid flowing in an eccentric annulus. Szabo and Hassager[3] and Bittleston and Walton[4] both present thoroughly studies on the flow of Bingham fluids in an eccentric annulus.

In this paper velocity profiles are presented from numerical solutions for the flow of several non-Newtonian fluids in an eccentric annulus. These velocity profiles are compared with experimentally obtained velocity profiles.

NUMERICAL METHOD

Basic equations

The basic equation for stationary axial flow of a non-Newtonian fluid in an eccentric annulus is

$$\nabla(\mu(\Pi)\nabla v)_z = -\frac{dp}{dz}, \qquad (1)$$

with the proper boundary conditions. Were dp/dz denotes the pressure gradient, μ is the shear rate dependent viscosity and Π the invariant of the shear rate tensor is defined as

$$\Pi = \sqrt{\tfrac{(\dot{\gamma}:\dot{\gamma})}{2}} \qquad (2)$$

with $\dot{\gamma}$ as the shear rate. The boundary conditions is noslip on the walls. The bipolar co-ordinate transformation[5] maps the eccentric annulus onto a square domain. The Finite Element Method (FEM)[6] is used to solve Eqn. 1 over this square domain using second order quadrilateral elements[7].

The non linearity arising from the viscosity function is linearised by Picard iteration.

To facilitate the comparison with the experimental data the yield power law constitutive equation

$$\tau = \tau_y + A\dot{\gamma}^n \quad (3)$$

is used in this paper. Where τ_y is the yield value. A and n are rheological parameters. This gives the following viscosity function

$$\mu = \frac{\tau_y}{\dot{\gamma} + \dot{\gamma}_0} + A\dot{\gamma}^{n-1} \quad (4)$$

where an artificial shear rate $\dot{\gamma}_0$ is introduced to avoid numerical singularities when the shear rate tends to zero. The value of $\dot{\gamma}_0$ used is 10^{-8}.

It was taken advantage of the symmetries of the eccentric annulus and the calculational domain is as shown in Fig. 1. This also enables us to use a equation solver from the lapack library of linear algebra routines.

Numerical Error

The FEM error is expected to be proportional to the factor $(1/N_{ele})^2$ were N_{ele} is the number of elements, but this has not yet been verified.

The bipolar co-ordinate system is ideal for problems in an eccentric annulus Fig 1., but when the eccentricity tends to unity,

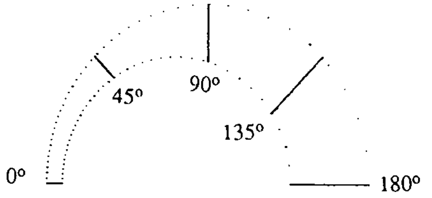

Figure 1. An eccentric annulus with the angular positions where the velocity profile has been measured.

the ratio between area of the largest and smallest element tends towards infinity. The error is dependent of this ratio, therefore the eccentricity schould not exceed δ=0.95.

RESULTS OF SIMULATIONS

To illustrate the severe effect of eccentricity on the velocity profile a simulation is preformed with δ=0.5. The velocities is normalized with the maximum velocity in the corresponding concentric case, i.e. with the same pressure gradient and rheological parameters. The diameter ratio of the inner and outer tube is 0.7

The power law constitutive equation is given by Eqn. 3. with τ_y=0. The value of the rheological parameter was A=0.25 and n=0.6. The pressure gradient used was 1442.0 Pa/m. The velocity profile is plotted in Fig. 2.

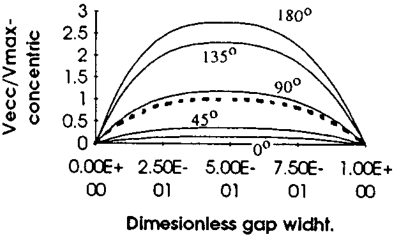

Figure 2. The velocity profile of a power law fluid. Solid line: eccentric annulus at different angles. Dotted line concentric annulus.

The ratio of the flow rates $R_Q = Q_{ecc}/Q_{con} = 1.59$. The effect of eccentricity is even more dramatic when considering the ratio of the maximum velocity in the wide and small annular gap $R_{vel} = V_{max,180^\circ}/V_{max,0^\circ} = 16.94$.

VERIFICATION

Data for verification is provided by Saga Petroleum A.S. Three different simulations have been done to be able to compare with the experimental data. Results of the first two simulations are presented in Fig.3.. The flowing fluid is a yield power law fluid at two different flow rates. The difference between the measured and calculated flowrate is less than 0.2 % for both the high and low flowrate. The difference between the calculated and measured maximum velocity in the wide annular gap is 5% for the high rate and 2% for the low rate.

In Fig 3. the calculated plug width is larger than the measured plug width. The calculated value is within 5% of the value that can be calculated by considering a force balance over the plug.

$$\Delta R_{plug} = 2\frac{\tau_y}{dp/dx} \quad (5)$$

This is reasonable because the numerical resolution in the radial direction is ±2%, and it may indicate that the yield point value accompanying the experimental velocity data is underestimated.

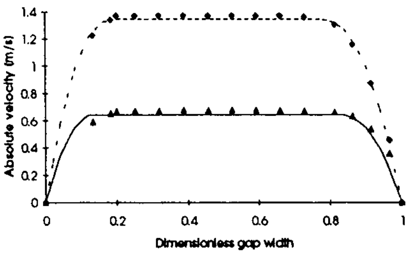

Figure 3. Comparison of the calculated velocity profile with the experimentally obtained velocity profile for $\delta=0.0$, i.e. a concentric annulus. Lines numerical results, dots are experimental data.

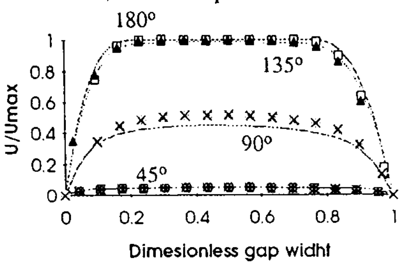

Figure 4. Comparison of the calculated velocity profile with the experimentally obtained velocity profile for $\delta=0.33$. Solid lines numerical results, dotted lines experimental data.

In Fig. 4. the calculated and measured velocity profile of a yield power law fluid flowing in an eccentric annulus with $\delta=0.33$. The velocity profile is plotted at 0°, 45°, 90° and 135°. At 180° no experimental data are given, the numerical calculations predicted a stagnant plug region for 180°. The angular positions are indicated in Fig. 1. The calculated volumetric rate differs with less than 0.1% from the measured volumetric rate, and the maximum velocity in the wide annular space deviates with less than 2.5% from the measured maximum velocity.

There are small deviations between the calculated and the measured velocity profile, notably at 90° were the velocity differs with 5%. This can be attributed to the fact that the flow rate when measuring the profile at 0°, was larger than the rate when they measured the profile at 90°. In addition the rheological parameters given may be slightly different from the actual fluid rheology as is indicated in the discussion of Fig. 3.

CONCLUSION

Numerical methods enable us to calculate the velocity profile of a generalised Newtonian fluid to a high degree of accuracy, as illustrated with the comparison with the experimental data.

The calculated profile schould enable us to derive related quantities such as pressure drop, viscosity field, shear rate field and tension on the wall with a high degree of accuracy.

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REFERENCES

1. T. L. Guckes, "Laminar Flow of Non-Newtonian Fluids in an Eccentric Annulus", J. Engineering for Industry, pp. 498-506, MAY 1975
2. M. Haciislamoglu and J. Langlinais, "Non- Newtonian Flow in Eccentric Annuli", J. Energy Resources Technology, Vol. 112, pp. 163-169, 1990.
3. P Szabo and O. Hassager, "Flow of Viscoplastic Fluids in Eccentric Annular Geometries", J. Non-Newtonian Fluid Mech., 45(2), pp. 149-169, 1992.
4. I.C. Walton and S.H. Bittleston, "The axial flow of a Bingham plastic in a narrow eccentric annulus", J. Fluid Mech., Vol. 222, pp. 39-60, 1991.
5. R.B. Bird, R.C. Armstrong and O. Hassager, "Dynamics of polymeric liquids", Vol 1, 2nd. ed., App. A, Wiley 1987.
6. C. Johnson, "Numerical solutions of partial differential equations by the finite element method", Studentlitteratur 1987.
7. C. Cuvelier, A. Segal and A. A. van Steenhoven, "Finite Element Methods and Navier-Stokes Equations", Reidel 1986.